KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF CHEMICAL ENGINEERING

THE DATA BOOK

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Dear student:
Please return this hardcopy to your instructor after the end of the exam.

Dear instructor:
If you are not teaching this course next semester, kindly return this hardcopy to the CHE secretary.

Preface

The aim of compiling this Data Book is to unify all core material into one place. Many a time, undergraduate students ask which formulae to commit to memory, as they approach exams. This Data Book contains all the key relations used frequently in core courses that tend to be longer, comprise subtle but important details, or are essential and hence necessitate repeat exposure for effective learning. Such a resource is a synopsis of the entire chemical engineering degree, therefore yielding a bird's eye view of how all the component parts work collectively with the benefit of minimal text often seen in conventional textbooks. The idea of the Data Book has proven to work effectively at other esteemed universities, with subject matter organized according to individual topics. While no compilation is perfect, we have aimed to strike a balance such that not every conceivable relation need be listed, thereby facilitating the student's ability to efficiently look up the needed information. We hope such a Data Book will serve as a useful study aid during their time within the department.

Prepared by R.S.M. Chrystie *et al.* (2022) ©

Acknowledgement

We are very grateful to all the chemical engineering faculty in assisting the compilation of this degree-wide formula book.

PHYSICAL CONSTANTS

Avogadro's constant	N_A	6.022×10^{23}	mol^{-1}
Boltzmann's constant	k_B	1.381×10^{-23}	J/K
Charge on electron	e	1.602×10^{-19}	C
Gravitational acceleration	g	9.81	m/s^2
Mass of electron	m_e	9.110×10^{-31}	kg
Planck's constant	h	6.626×10^{-34}	J s
Standard pressure	P_o	1.013×10^{5}	Pa
Standard temperature	T_o	273.15	K
Stefan-Boltzmann constant	σ	5.670×10^{-8}	$W/(m^2 K^4)$
Universal gas constant	R	8.314	J/(mol K)
Velocity of light in vacuum	c	2.998×10^{8}	m/s
Volume of an ideal gas at STP	V_o	2.241×10^{-2}	m^3/mol

GREEK ALPHABET

A	α	alpha	N	ν	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	Υ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

SYMBOL USE

Alternative notation in textbooks

Specific/molar value (J/kg) or (J/mol)	*Absolute value* (J)	*Partial value* (J/mol)
$\hat{H}$	H	
H	H^t	$\overline{H}_i$
h	H	

Our convention in this book

Every effort is made here to use a consistent and user-friendly set of symbols.

Beware of similar symbols

h	*Planck's constant*
h	*heat transfer coefficient*
h	*height*
h	*fluid head*
H	*specific enthalpy*
h	*interval width*
h_{fg}	*latent heat*
H	*Henry's constant*

CONTEXT IS IMPORTANT

UNIT CONVERSION

Mechanical

Quantity	SI			Additional (non-SI)		
	Name	**Symbol**	**Definition**	**Name**	**Symbol**	**Definition**
Force	Newton	N	kg m s^{-2}	dyne	dyn	g cm s^{-2}
Torque		N m				
Work, Energy	Joule	J	N m	erg	erg	10^{-7} J
Power	Watt	W	J s^{-1}			
Pressure	Pascal	Pa	N m^{-2}	bar	bar	10^5 Pa
Stress	Pascal	Pa	N m^{-2}			
Dynamic viscosity		Pa s		poise	P	g $cm^{-1}s^{-1}$
Kinematic viscosity		m^2 s^{-1}		stokes	St	cm^2 s^{-1}

Thermal

Quantity	SI			Additional (non-SI)		
	Name	**Symbol**	**Definition**	**Name**	**Symbol**	**Definition**
Temperature	Kelvin	K		rankine	°R	$T_R = 1.8T_K$
Energy, Work, Heat	Joule	J	N m	kilowatt-hour	kWh	3.6 MJ
Specific heat capacity		J $kg^{-1}K^{-1}$				
Specific entropy		J $kg^{-1}K^{-1}$				
Thermal conductivity		W $m^{-1}K^{-1}$				
Heat transfer coefficient		W $m^{-2}K^{-1}$				
Surface tension		N m^{-1}				

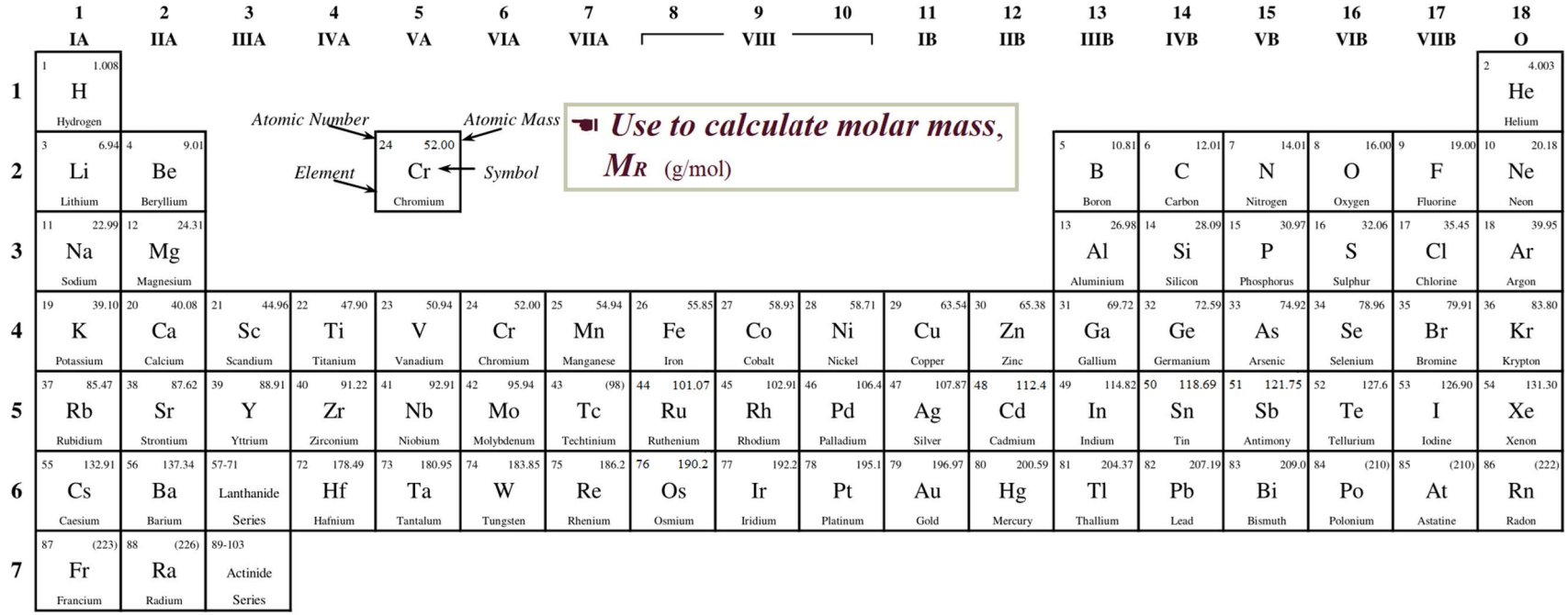
Electrical

Quantity	SI			Additional (non-SI)		
	Name	**Symbol**	**Definition**	**Name**	**Symbol**	**Definition**
Energy	Joule	J	N m	kilowatt-hour electronvolt	kWh eV	3.6 MJ 0.1602 aJ
Power	Watt	W	J s^{-1}			
Current	Ampere	A				
Charge	Coulomb	C	A s			
Potential, e.m.f.	Volt	V				
Resistance	Ohm	Ω	V A^{-1}			

DIMENSIONLESS GROUPS

Drag Coefficient	$C_D = \frac{2F_D}{\rho v^2 A}$	Nusselt	$Nu = \frac{hL}{k}$
Bond	$Bo = \frac{\Delta\rho g L^2}{\sigma}$	Peclet (Heat)	$Pe = RePr = \frac{c_p \rho v L}{k}$
Flow Coefficient	$C_Q = \frac{\dot{V}}{N D^3}$	Peclet (Mass)	$Pe' = ReSc = \frac{vL}{\mathfrak{D}}$
Fourier	$Fo = \frac{kt}{\rho c_p L^2}$	Power Coefficient	$N_P = \frac{\mathbb{P}}{\rho N^3 D^5}$
Fanning Friction	$C_f = \frac{2\tau_w}{\rho v^2}$	Prandtl	$Pr = \frac{\mu c_p}{k}$
Froude	$Fr = \frac{v}{\sqrt{gh}}$	Rayleigh	$Ra = GrPr = \frac{L^3 \rho g \Delta\rho c_p}{\mu k}$
Grashof	$Gr = \frac{L^3 \rho g \Delta\rho}{\mu^2}$	Reynolds	$Re = \frac{\rho v L}{\mu}$
Head Coefficient	$C_H = \frac{gh}{N^2 D^2}$	Richardson	$Ri = \frac{Gr}{Re^2} = \frac{gL\Delta\rho}{\rho v^2}$
j-factor (Heat)	$j_H = St\ Pr^{2/3}$	Schmidt	$Sc = \frac{\mu}{\rho\mathfrak{D}}$
j-factor (Mass)	$j_D = St'Sc^{2/3}$	Sherwood	$Sh = \frac{k_c L}{\mathfrak{D}}$
Lewis	$Le = \frac{Sc}{Pr} = \frac{k}{\rho c_p \mathfrak{D}}$	Specific Speed	$N_S = \frac{N\sqrt{\dot{V}}}{(gh)^{3/4}}$
Mach	$M = \frac{v}{a}$	Stanton	$St = \frac{Nu}{RePr} = \frac{h}{\rho v c_p}$
Morton	$Mo = \frac{\mu^4 g \Delta\rho}{\rho^2 \sigma^3}$	Modified Stanton	$St' = \frac{Sh}{ReSc} = \frac{k_c}{v}$
		Weber	$We = \frac{\rho v^2 L}{\sigma}$

PERIODIC TABLE



	1 IA	2 IIA	3 IIIA	4 IVA	5 VA	6 VIA	7 VIIA	8	9 VIII	10	11 IB	12 IIB	13 IIIB	14 IVB	15 VB	16 VIB	17 VIIB	18 O
1	1 1.008 H Hydrogen																	2 4.003 He Helium
2	3 6.94 Li Lithium	4 9.01 Be Beryllium											5 10.81 B Boron	6 12.01 C Carbon	7 14.01 N Nitrogen	8 16.00 O Oxygen	9 19.00 F Fluorine	10 20.18 Ne Neon
3	11 22.99 Na Sodium	12 24.31 Mg Magnesium											13 26.98 Al Aluminium	14 28.09 Si Silicon	15 30.97 P Phosphorus	16 32.06 S Sulphur	17 35.45 Cl Chlorine	18 39.95 Ar Argon
4	19 39.10 K Potassium	20 40.08 Ca Calcium	21 44.96 Sc Scandium	22 47.90 Ti Titanium	23 50.94 V Vanadium	24 52.00 Cr Chromium	25 54.94 Mn Manganese	26 55.85 Fe Iron	27 58.93 Co Cobalt	28 58.71 Ni Nickel	29 63.54 Cu Copper	30 65.38 Zn Zinc	31 69.72 Ga Gallium	32 72.59 Ge Germanium	33 74.92 As Arsenic	34 78.96 Se Selenium	35 79.91 Br Bromine	36 83.80 Kr Krypton
5	37 85.47 Rb Rubidium	38 87.62 Sr Strontium	39 88.91 Y Yttrium	40 91.22 Zr Zirconium	41 92.91 Nb Niobium	42 95.94 Mo Molybdenum	43 (98) Tc Techtinium	44 101.07 Ru Ruthenium	45 102.91 Rh Rhodium	46 106.4 Pd Palladium	47 107.87 Ag Silver	48 112.4 Cd Cadmium	49 114.82 In Indium	50 118.69 Sn Tin	51 121.75 Sb Antimony	52 127.6 Te Tellurium	53 126.90 I Iodine	54 131.30 Xe Xenon
6	55 132.91 Cs Caesium	56 137.34 Ba Barium	57-71 Lanthanide Series	72 178.49 Hf Hafnium	73 180.95 Ta Tantalum	74 183.85 W Tungsten	75 186.2 Re Rhenium	76 190.2 Os Osmium	77 192.2 Ir Iridium	78 195.1 Pt Platinum	79 196.97 Au Gold	80 200.59 Hg Mercury	81 204.37 Tl Thallium	82 207.19 Pb Lead	83 209.0 Bi Bismuth	84 (210) Po Polonium	85 (210) At Astatine	86 (222) Rn Radon
7	87 (223) Fr Francium	88 (226) Ra Radium	89-103 Actinide Series															

Lanthanides	57 138.91 La Lanthanum	58 140.12 Ce Cerium	59 140.91 Pr Praseodymium	60 144.24 Nd Neodymium	61 146.92 Pm Promethium	62 150.53 Sm Samarium	63 151.96 Eu Europium	64 157.25 Gd Gadolinium	65 158.92 Tb Terbium	66 162.50 Dy Dysprosium	67 164.93 Ho Holmium	68 167.26 Er Erbium	69 168.93 Tm Thulium	70 170.34 Yb Ytterbium	71 174.97 Lu Lutetium
Actinides	89 227.03 Ac Actinium	90 232.04 Th Thorium	91 (231) Pa Protactinium	92 238.03 U Uranium	93 (237) Np Neptunium	94 (239) Pu Plutonium	95 (241) Am Americium	96 (247) Cm Curium	97 (249) Bk Berkelium	98 (251) Cf Californium	99 (254) Es Einsteinium	100 (257) Fm Fermium	101 (258) Md Mendelevium	102 (255) No Nobelium	103 (257) Lw Lawrencium

KINETICS & REACTORS

Species balancing

Conversion of mole and mass fractions:

$$w_A = \frac{x_A M_A}{x_A M_A + x_B M_B + \cdots} \qquad x_A = \frac{w_A / M_A}{w_A / M_A + w_B / M_B + \cdots}$$

w_A: mass fraction of A
x_A: mole fraction of A
M_A: molar mass of A

Most used formula in CHE:

"IN + GEN = OUT + ACC"

$$F_{j\,in} + G_j = F_{j\,out} + \frac{dN_j}{dt}$$

Where: $G_j = r_j V$ for species j, *noting:* $G_j > 0$ (production)

$G_j < 0$ (consumption)

Single generic reaction:

$$A + \frac{b}{a}B \rightleftharpoons \frac{c}{a}C + \frac{d}{a}D$$ with forward rate k_f and reverse rate k_r

The basis here assumes A to be the limiting reactant.

Fractional conversion (*based on A*): $X_A \equiv \frac{\text{moles A reacted}}{\text{moles A fed}} \Longrightarrow F_A = F_{A0}(1 - X_A)$

$$\downarrow$$

$$F_j = F_{j0} + F_{A0} X_A \nu_j$$

where: $-r_A = \frac{r_j}{\nu_j}$ $\nu_B = -\frac{b}{a}$ $\nu_A = -1$ $\nu_C = \frac{c}{a}$ $\nu_D = \frac{d}{a}$

Extent of reaction (*dimensionless*): $\xi \equiv \frac{\text{moles } j \text{ reacted}}{\text{stoichiometry}} \implies F_j = F_{j0} + \xi \nu_j$

Reaction thermodynamics

$$k(T) = \mathbb{A} e^{-\frac{E_a}{RT}}$$

Arrhenius

$$\ln \frac{K_p(T)}{K_p(T_1)} = \frac{\Delta H_r^o}{R}\left[\frac{1}{T_1} - \frac{1}{T}\right]$$

Van't Hoff

$$K_c = \frac{k_f}{k_r} \qquad K_c = \frac{C_C^{c/a}\, C_D^{d/a}}{C_A\ C_B^{b/a}} \qquad K_p = \frac{P_C^{c/a}\, P_D^{d/a}}{P_A\ P_B^{b/a}}$$

$$\delta = \frac{c}{a} + \frac{d}{a} - \frac{b}{a} - 1 \qquad K_p = K_c (RT)^{\delta}$$

$$-r_A = k_f C_A^{\alpha} C_B^{\beta} - k_r C_C^{\gamma} C_D^{\varepsilon}$$

Rate Law, whose overall order is $\alpha + \beta$ (forward direction)
Net reaction rate for A

Energy balance for open systems:

$$\boxed{\dot{Q} + \dot{W}_s = \Delta \dot{H} + \frac{d E_{sys}}{dt}}$$

Where: $\Delta \dot{H} = \dot{\xi}\, \Delta H_r(T) + \sum F_j\, H_j - \sum F_{j0}\, H_{j0}$

With: $\Delta H_r(T) = \Delta H_r^o\left(T_{ref}\right) + \left(T - T_{ref}\right) \sum \nu_j c_{p,j}$

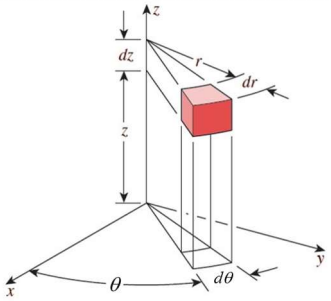
FUNDAMENTAL GEOMETRY

Cylindrical coordinates

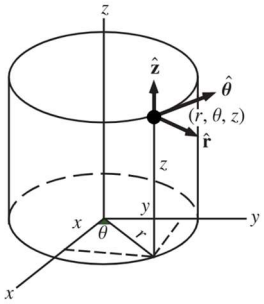
(r, θ, z)

θ is the azimuthal angle

Note: both coordinate systems here follow the convention used in mathematics



← A differential volume element in cylindrical coordinates

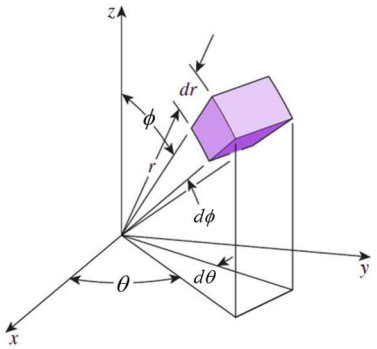


$x = r\cos\theta$ $\quad y = r\sin\theta$ $\quad r = \sqrt{x^2 + y^2}$ $\quad z = z$

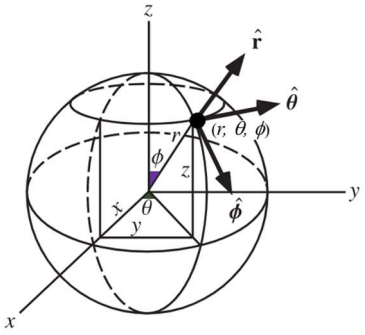
Spherical coordinates

(r, θ, ϕ)

θ is the azimuthal angle
ϕ is the zenith angle

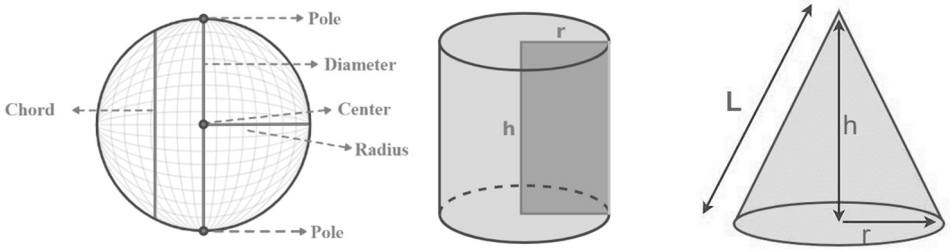


← A differential volume element in spherical coordinates



$x = r\cos\theta\sin\phi$ $\quad y = r\sin\theta\sin\phi$ $\quad r = \sqrt{x^2 + y^2 + z^2}$ $\quad z = r\cos\phi$

Basic shapes



Total surface area:	$4\pi r^2$	$2\pi r^2 + 2\pi rh$	$\pi r(r + L)$
Volume:	$\frac{4}{3}\pi r^3$	$\pi r^2 h$	$\frac{1}{3}\pi r^2 h$

AVERAGING

Weighted average (discrete)

$$\overline{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Ordinary average (continuous)

$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Note the weighted average becomes the ordinary average
when: $w_i = 1$ for all i

POLYNOMIALS

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Partial fraction decomposition

$$f(x) = \frac{P(x)}{Q(x)}$$ with the order of $P < Q$ (see example below)

$$\frac{2x^6 - 4x^5 + 5x^4 - 3x^3 + x^2 + 3x}{(x-1)^3\left(x^2+1\right)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{\left(x^2+1\right)^2}$$

Solve $A \to G$ by comparing coefficients of x

TRIGONOMETRIC FORMULAE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \qquad \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sinh x = -i\sin ix = \tfrac{1}{2}(e^x - e^{-x}) \qquad \cosh x = \cos ix = \tfrac{1}{2}(e^x + e^{-x})$$

DIFFERENTIATION

For functions $u(x)$ and $v(x)$:

Product Rule

$$\frac{d(uv)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain rule

When $x, y, z, \ldots$ are functions of $u, v, w, \ldots$

$$\left(\frac{\partial\phi}{\partial u}\right)_{v,w,\ldots} = \frac{\partial\phi}{\partial x}\left(\frac{\partial x}{\partial u}\right)_{v,w,\ldots} + \frac{\partial\phi}{\partial y}\left(\frac{\partial y}{\partial u}\right)_{v,w,\ldots} + \frac{\partial\phi}{\partial z}\left(\frac{\partial z}{\partial u}\right)_{v,w,\ldots} + \cdots$$

Total derivative

For any function $\phi(x, y, z, \ldots)$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz + \cdots$$

in which $\frac{\partial\phi}{\partial x}$ means $\left(\frac{\partial\phi}{\partial x}\right)_{y,z,\ldots}$ (i.e. with $y, z, \ldots$ kept constant)

If $f(x, y)dx + g(x, y)dy = d\phi$ for some function $\phi(x, y)$, then $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

INTEGRATION

Integration by parts

$$\int u(x)\frac{dv(x)}{dx}\,dx = u(x)v(x) - \int v(x)\frac{du(x)}{dx}\,dx$$

for a product of two functions $u(x)$ and $v(x)$

Logarithmic integrals

$$\int \frac{1}{1+\varepsilon x}\,dx = \frac{1}{\varepsilon}\ln(1+\varepsilon x) + \mathbb{C}$$

$$\int \frac{1+\varepsilon x}{(1-x)^2}\,dx = \frac{(1+\varepsilon)x}{1-x} - \varepsilon \ln\frac{1}{1-x} + \mathbb{C}$$

$$\int \frac{(1+\varepsilon x)^2}{(1-x)^2}\,dx = 2\varepsilon(1+\varepsilon)\ln(1-x) + \varepsilon^2 x + \frac{(1+\varepsilon)^2 x}{1-x} + \mathbb{C}$$

Trigonometric & hyperbolic integrals

$$\int \cos x\,dx = \sin x + \mathbb{C}$$

$$\int \cosh x\,dx = \sinh x + \mathbb{C}$$

$$\int \tan x\,dx = -\ln(\cos x) + \mathbb{C}$$

$$\int \tanh x\,dx = \ln(\cosh x) + \mathbb{C}$$

$$\int \operatorname{cosec} x\,dx = \ln\left(\tan\tfrac{x}{2}\right) + \mathbb{C}$$

$$\int \operatorname{cosech} x\,dx = \ln\left(\tanh\tfrac{x}{2}\right) + \mathbb{C}$$

$$\int \sec x\,dx = \ln(\tan x + \sec x) + \mathbb{C}$$

$$\int \operatorname{sech} x\,dx = 2\tan^{-1}(e^x) + \mathbb{C}$$

$$\int \cot x\,dx = \ln(\sin x) + \mathbb{C}$$

$$\int \coth x\,dx = \ln(\sinh x) + \mathbb{C}$$

$$\int \sec^2 x\,dx = \tan x + \mathbb{C}$$

$$\int \operatorname{sech}^2 x\,dx = \tanh x + \mathbb{C}$$

$$\int \tan x \sec x\,dx = \sec x + \mathbb{C}$$

$$\int \tanh x \operatorname{sech} x\,dx = -\operatorname{sech} x + \mathbb{C}$$

$$\int \cot x \operatorname{cosec} x\,dx = -\operatorname{cosec} x + \mathbb{C}$$

$$\int \coth x \operatorname{cosech} x\,dx = -\operatorname{cosech} x + \mathbb{C}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + \mathbb{C}$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + \mathbb{C}$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \mathbb{C}$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + \mathbb{C}$$

$$\int \frac{1}{a+bx^4}\,dx = \frac{k}{4a}\left[\ln\left|\frac{x+k}{x-k}\right| + 2\tan^{-1}\left(\frac{x}{k}\right)\right] + \mathbb{C}$$

valid for $ab < 0$, where $k = \sqrt[4]{-\frac{a}{b}}$

Standard substitutions

If the integrand is a function of:	Substitute
$(a^2 - x^2)$ or $\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
$(a^2 + x^2)$ or $\sqrt{a^2 + x^2}$	$x = a\tan\theta$ or $x = a\sinh\theta$
$(x^2 - a^2)$ or $\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $x = a\cosh\theta$

If the integral is of the form:	Substitute
$\int \frac{dx}{(ax+b)\sqrt{px+q}}$	$px + q = u^2$
$\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$	$ax + b = \frac{1}{u}$

DIFFERENTIAL EQUATIONS

First-order linear ODE

A first-order linear ODE of the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

can be solved by using the integrating factor $e^{\int P\,dx}$, such that:

$$\frac{d}{dx}\left[ye^{\int P\,dx}\right] = Q(x)e^{\int P\,dx}$$

Second-order linear ODE

A second-order linear ODE with *constant coefficients* of the form:

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

can be solved by adding together the complementary function with the particular solution, such that:

$$y(x) = y_{CF} + y_{PS}$$

First, solve the auxiliary equation, such that:

$$a\,m^2 + b\,m + c = 0$$

Whose roots are m_1 and m_2

Root	y_{CF}
Real $m_1 \neq m_2$	$Ae^{m_1 x} + Be^{m_2 x}$
Real $m_1 = m_2$	$(A + Bx)e^{m_1 x}$
$m = p \pm qi$	$e^{px}(A\cos qx + B\sin qx)$

Second, find a form of y_{PS} according to RHS of the ODE:

$f(x)$	y_{PS}
k (a constant)	C
Linear in x	$Cx + D$
Quadratic in x	$Cx^2 + Dx + E$
$k\sin px$ or $k\cos px$	$C\cos px + D\sin px$
ke^{px}	Ce^{px}
Sum of the above	Sum of the above
Product of the above	Product of the above

Note: If suggested form of y_{PS} already appears in the complementary function, then multiply suggested form by x.

STATIONARY POINTS

Unconstrained

Stationary points occur for $f(x, y)$ where $\nabla f = 0$

i.e. where $\dfrac{\partial f}{\partial x} = 0$ and $\dfrac{\partial f}{\partial y} = 0$ simultaneously

Let (a, b) be the stationary point and define:

$$f_{xx} = \left.\frac{\partial^2 f}{\partial x^2}\right|_{a,b} \qquad f_{yy} = \left.\frac{\partial^2 f}{\partial y^2}\right|_{a,b} \qquad f_{xy} = \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{a,b}$$

If $f_{xy}^2 - f_{xx} f_{yy} < 0$ and $f_{xx} < 0$ then $f(x, y)$ has a maximum at (a, b)

If $f_{xy}^2 - f_{xx} f_{yy} < 0$ and $f_{xx} > 0$ then $f(x, y)$ has a minimum at (a, b)

If $f_{xy}^2 - f_{xx} f_{yy} > 0$ then $f(x, y)$ has a saddle point at (a, b)

If $f_{xy}^2 - f_{xx} f_{yy} = 0$ then the nature of the turning point depends on higher order derivatives

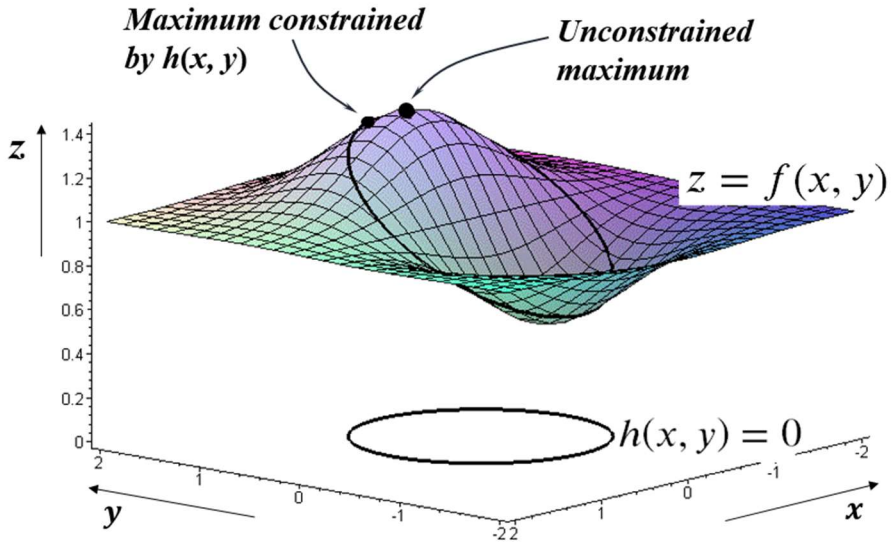
Constrained

Lagrange's method of undetermined multipliers:

Stationary points for $f(x, y)$ along the line $h(x, y) = 0$ are coincident with the stationary points for $L(x, y, \lambda)$, where:

$$L(x, y, \lambda) = f(x, y) - \lambda\, h(x, y)$$

i.e. where $\dfrac{\partial L}{\partial x} = 0$, $\dfrac{\partial L}{\partial y} = 0$ and $\dfrac{\partial L}{\partial \lambda} = h(x, y) = 0$ simultaneously



NUMERICAL METHODS

Solving equations

Newton Raphson $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Linear regression

Straight line through scatterplot: $\quad y = a_0 + a_1 x$

Where: $\quad a_1 = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$ $\qquad$ and $\qquad a_0 = \overline{y} - a_1 \overline{x}$

Numerical differentiation

y_n are values of y at equal intervals of x with width h

$$\left.\frac{dy}{dx}\right]_n = \frac{y_{n+1} - y_{n-1}}{2h} + O(h^2) \qquad \left.\frac{dy}{dx}\right]_n = \frac{-3y_n + 4y_{n+1} - y_{n+2}}{2h} + O(h^2)$$

$$\left.\frac{d^2y}{dx^2}\right]_n = \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} + O(h^2)$$

Differential equations

Problem:	$\dfrac{dy(x)}{dx} = f(x, y), \qquad y_0 = y(x_0)$
Euler method:	$y_{i+1} = y_i + h\, f(x_i, y_i)$

Numerical integration

Trapezium Rule (N is the number of intervals)

$$\int_{x_0}^{x_N} y\, dx = \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_n + \cdots + 2y_{N-1} + y_N)$$

Simpsons's Rule (N is the number of intervals, which must be even)

$$\int_{x_0}^{x_N} y\, dx = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-1} + 4y_n + 2y_{n+1} + \cdots + 4y_{N-1} + y_N\right)$$

DYNAMICS & CONTROL

For a feedback loop, the ideal PID (Proportional-Integral-Derivative) control law is given by:

$$p(t) - \overline{p} = K_c \left[e(t) + \frac{1}{\tau_I} \int_0^t e(\theta)\, d\theta + \tau_D \frac{de(t)}{dt} \right]$$

Name	$G(s)$	**Amplitude Ratio** $AR(\omega)$	**Phase Shift** $\phi(\omega)$
First order lag	$\frac{1}{\tau s + 1}$	$\frac{1}{\sqrt{\tau^2\omega^2 + 1}}$	$-\tan^{-1}\tau\omega$
First order lead	$\tau s + 1$	$\sqrt{\tau^2\omega^2 + 1}$	$\tan^{-1}\tau\omega$
Integrator	$\frac{1}{\tau_I s}$	$\frac{1}{\tau_I \omega}$	$-\frac{\pi}{2}$
Differentiator	$\tau_D s$	$\tau_D \omega$	$\frac{\pi}{2}$
Dead time	$e^{-t_d s}$	1	$-t_d\omega$

LAPLACE TRANSFORMS

$$\mathscr{L}\{y(t)\} \equiv Y(s) = \int_0^\infty y(t) e^{-st}\, dt$$

Simple functions

$y(t)$	$Y(s)$	$y(t)$	$Y(s)$
1	$\frac{1}{s}$	$\cos(\omega t + \theta)$	$\frac{s\cos\theta - \omega\sin\theta}{s^2 + \omega^2}$
t	$\frac{1}{s^2}$	$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2 + \omega^2}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\cos\omega t$	$\frac{s+\alpha}{(s+\alpha)^2 + \omega^2}$

$\sin(\omega t + \theta)$	$\frac{s\sin\theta + \omega\cos\theta}{s^2 + \omega^2}$	$\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)$	$\frac{e^{-k\sqrt{s}}}{s}$

Derivatives & Integrals

$y(t)$	$Y(s)$	$y(t)$	$Y(s)$
$\frac{dx(t)}{dt}$	$sX(s) - x(0)$	$\int_0^t x(\tau)\,d\tau$	$\frac{X(s)}{s}$
$t\,x(t)$	$-\frac{dX(s)}{ds}$	$\frac{x(t)}{t}$	$\int_s^\infty X(\sigma)\,d\sigma$
$\frac{d^n x(t)}{dt^n}$	$s^n X(s) - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - x^{(n-1)}(0)$		
$\int_0^t x_1(\tau)x_2(t-\tau)d\tau$	$X_1(s)X_2(s)$		

Step & Impulse Functions

$y(t)$	$Y(s)$	$y(t)$	$Y(s)$
$H(t)$	$\frac{1}{s}$	$H(t-\tau)$	$\frac{e^{-s\tau}}{s}$
$\delta(t)$	1	$\delta(t-\tau)$	$e^{-s\tau}$

General Properties

$y(t)$	$Y(s)$	$y(t)$	$Y(s)$
$H(t-\tau)x(t-\tau)$	$e^{-s\tau}X(s)$	$e^{-\alpha t}x(t)$	$X(s+\alpha)$
$x(\alpha t)$	$\frac{X(s/\alpha)}{\alpha}$	$\delta(t-\tau)$	$e^{-s\tau}$

$$\lim_{t\to 0} x(t) = \lim_{s\to\infty} sX(s) \qquad \lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

VECTORS & MATRICES

Matrix properties

$$(\mathbf{AB}\ldots\mathbf{X})^{\mathbf{T}} = \mathbf{X}^{\mathbf{T}}\ldots\mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}$$ Reversal rule

$$(\mathbf{AB}\ldots\mathbf{X})^{-1} = \mathbf{X}^{-1}\ldots\mathbf{B}^{-1}\mathbf{A}^{-1}$$ (if inverses exist)

$\mathbf{A}\,\boldsymbol{x} = \boldsymbol{b}$ can be solved for vector $\boldsymbol{x}$ if $\mathbf{A}$ is square, $\mathbf{A} \neq 0$, and $det\mathbf{A} \neq 0$

$$\mathbf{K}' = \mathbf{R}\,\mathbf{K}\,\mathbf{R}^{\mathbf{T}}$$ where transformation matrix $\mathbf{R}$ rotates *property matrix* $\mathbf{K}$ into another coordinate system yielding $\mathbf{K}'$

Vector algebra

Cross product (*right hand rule*)

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Scalar triple product

$$\boldsymbol{a} \bullet (\boldsymbol{b} \times \boldsymbol{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{cases} (\boldsymbol{b} \times \boldsymbol{c}) \bullet \boldsymbol{a} \\ (\boldsymbol{a} \times \boldsymbol{b}) \bullet \boldsymbol{c} \\ (\boldsymbol{c} \times \boldsymbol{a}) \bullet \boldsymbol{b} \end{cases}$$

Vector triple product

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \bullet \mathbf{c})\boldsymbol{b} - (\boldsymbol{a} \bullet \mathbf{b})\boldsymbol{c}$$
$$(\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c} = (\boldsymbol{a} \bullet \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{b} \bullet \boldsymbol{c})\boldsymbol{a}$$

Vector calculus

$T(x, y, z)$ denotes a scalar function, and $\boldsymbol{q}(x, y, z)$ a vector function

$$\boldsymbol{q}(x, y, z) = q_x(x, y, z)\,\hat{\boldsymbol{x}} + q_y(x, y, z)\,\hat{\boldsymbol{y}} + q_z(x, y, z)\,\hat{\boldsymbol{z}} = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix}$$

Gradient

$$\text{grad}\, T = \nabla T = \frac{\partial T}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial T}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial T}{\partial z}\hat{\boldsymbol{z}}$$

Divergence $\quad \operatorname{div} \boldsymbol{q} = \nabla \bullet \boldsymbol{q} = \dfrac{\partial q_x}{\partial x} + \dfrac{\partial q_y}{\partial y} + \dfrac{\partial q_z}{\partial z}$

Curl $\quad \operatorname{curl} \boldsymbol{q} = \nabla \times \boldsymbol{q} = \left(\dfrac{\partial q_z}{\partial y} - \dfrac{\partial q_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\dfrac{\partial q_x}{\partial z} - \dfrac{\partial q_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\dfrac{\partial q_y}{\partial x} - \dfrac{\partial q_x}{\partial y}\right)\hat{\boldsymbol{z}}$

Laplacian $\quad \nabla \bullet \nabla T = \nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}$

$\nabla \bullet \nabla \boldsymbol{q} = \nabla^2 \boldsymbol{q} = \dfrac{\partial^2 \boldsymbol{q}}{\partial x^2} + \dfrac{\partial^2 \boldsymbol{q}}{\partial y^2} + \dfrac{\partial^2 \boldsymbol{q}}{\partial z^2}$

Advection operator $\quad (\boldsymbol{q} \bullet \nabla)\, \boldsymbol{b} = q_x \dfrac{\partial \boldsymbol{b}}{\partial x} + q_y \dfrac{\partial \boldsymbol{b}}{\partial y} + q_z \dfrac{\partial \boldsymbol{b}}{\partial z}$

Surface integral $\quad \dot{Q} = \iint_S \boldsymbol{q} \bullet d\boldsymbol{S}$ total flux of $\boldsymbol{q}$ through surface S

THERMODYNAMICS

Equations of state

Ideal gas law

where $\hat{R} = R/M_R$

$$\begin{cases} PV^t = nRT \\ PV^t = m\hat{R}T \\ PV = \hat{R}T \\ P = \rho\hat{R}T \end{cases}$$

Van der Waals

$$P = \frac{RT}{V-b} - \frac{a}{V^2} \qquad a = \frac{27}{64}\frac{R^2T_c^2}{P_c} \qquad b = \frac{1}{8}\frac{RT_c}{P_c}$$

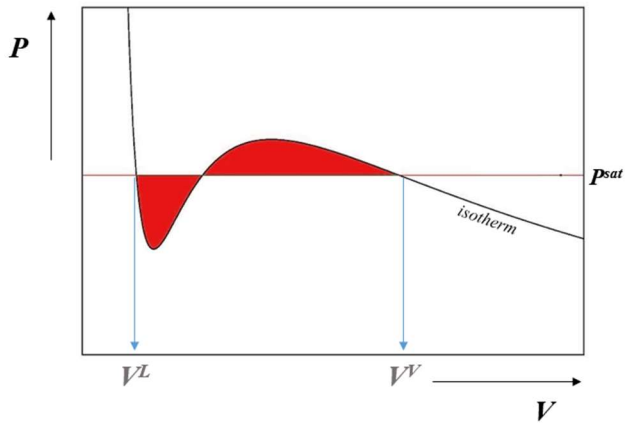
Update Formulae

Liquid root $\rightarrow V^L$

Initial guess: $V = b$	$V_{n+1} = b + V_n^2 \left[\frac{RT + P(b - V_n)}{a}\right]$

Vapour root $\rightarrow V^V$

Initial guess: $V = \frac{RT}{P}$	$V_{n+1} = \frac{RT}{P} + b - \frac{a}{P}\frac{V_n - b}{V_n^2}$



$$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

Volume expansivity

$$\kappa \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

Isothermal compressibility

$$\ln P^{sat} = A - \frac{B}{T + C}$$

Antoine

$$\frac{dP^{sat}}{dT} = \frac{\Delta H_{vap}}{T \Delta V_{vap}}$$

Clapeyron

Perfect & ideal gases

Specific heat relationship	$c_p^{ig} - c_v^{ig} = \hat{R}$
Change in internal energy	$U_2 - U_1 = c_v^{ig}(T_2 - T_1)$
Change in enthalpy	$H_2 - H_1 = c_p^{ig}(T_2 - T_1)$

Heat capacity ratio

$$\gamma \equiv \frac{c_p^{ig}}{c_v^{ig}}$$

Speed of sound

$$a = \sqrt{\gamma \hat{R} T}$$

For *Isentropic* changes

$$PV^{\gamma} = const.$$
$$TV^{\gamma-1} = const.$$
$$T/P^{(\gamma-1)/\gamma} = const.$$

Fundamental relations

$$\Delta U = Q + W$$
Closed systems

$$\Delta \dot{H} = \dot{Q} + \dot{W}_s$$
Open systems (steady state)

Idealized reversible displacement work:

$$W_{rev} = -\int_{V_1}^{V_2} P \ dV$$

$$c_v \equiv \left(\frac{\partial U}{\partial T}\right)_v$$

$$c_p \equiv \left(\frac{\partial H}{\partial T}\right)_p$$

$$H \equiv U + PV$$
Enthalpy

$$A \equiv U - TS$$
Helmholtz

$$G \equiv H - TS$$
Gibbs

$$dU = TdS - PdV$$
$$dH = TdS + VdP$$
$$dG = VdP - SdT$$
$$dA = -SdT - PdV$$

Maxwell relations:

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

Real pure fluids

Compressibility factor: $$Z \equiv \frac{PV}{RT} = \frac{V}{V^{ig}}$$

Residual properties: $$M^R \equiv M - M^{ig}$$ where: M is either $V, U, H, S, A, or\ G$

Fugacity of pure species:

For all phases $$\ln\left(\frac{f}{P}\right) = \frac{1}{RT}\int_0^P \left(V - \frac{RT}{P}\right)\, dP$$

For a pure liquid $$\ln\left(\frac{f^{liq}}{f^{sat}}\right) = \frac{1}{RT}\int_{P^{sat}}^P V\, dP$$

Residual Gibbs-Free Energy:

$$G^R = RT\ \ln\phi \qquad \text{with} \quad \phi = {}^{f}\!/_{P}$$

$$G^R = A^R + PV - RT(1 + \ln Z) \qquad \text{with} \quad A^R = -\int_\infty^V \left(P - \frac{RT}{V}\right)\, dV$$

Mixtures

$$y_i P = x_i \gamma_i P_i^{sat}$$

Modified Raoult's Law

$$\alpha_{12} = \frac{y_1/x_1}{y_2/x_2}$$

Relative volatility

$$P = \sum_{i=1}^{N} x_i \gamma_i P_i^{sat}$$

Bubble point

$$P = \left[\sum_{i=1}^{N} \frac{y_i}{\gamma_i P_i^{sat}}\right]^{-1}$$

Dew point

$$\overline{M}_i \equiv \left(\frac{\partial M^t}{\partial n_i}\right)_{T,P,n_{j\neq i}}$$

Partial property

$$M = \sum_{i=1}^{N} x_i \overline{M}_i$$

Summation

Gibbs-Duhem $(const.\, T, P)$

$$\sum_{i=1}^{N} x_i \, d\overline{M}_i = 0$$

Properties at infinite dilution:

$$\lim_{x_i \to 0} \overline{M}_i = \overline{M}_i^{\infty}$$

$$\lim_{x_i \to 0} \gamma_i(T, P, x_i) = \gamma_i^{\infty}(T, P)$$

Excess properties: $M^E \equiv M - M^{id}$

where:
M is either $V, U, H, S, A, or\ G$

Activity coefficient: $RT \ln \gamma_i = \overline{G}_i^E$

TRANSPORT PROCESSES

Overview

Navier-Stokes:	$\rho\frac{D\boldsymbol{v}}{Dt} \equiv \rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\bullet\nabla\boldsymbol{v}\right) = -\nabla P + \mu\nabla^2\boldsymbol{v} + \boldsymbol{f}_b$
Energy Equation:	$\rho\frac{DT}{Dt} \equiv \rho\left(\frac{\partial T}{\partial t} + \boldsymbol{v}\bullet\nabla T\right) = \frac{k}{c_p}\nabla^2 T + \frac{\dot{e}_{gen}}{c_p}$
Continuity:	$\frac{\partial\rho}{\partial t} + \nabla\bullet(\rho\boldsymbol{v}) = 0$
Mass Transport:	$\rho\frac{DC_i}{Dt} \equiv \rho\left(\frac{\partial C_i}{\partial t} + \boldsymbol{v}\bullet\nabla C_i\right) = \rho\mathfrak{D}\nabla^2 C_i + \rho r_i$

FLUID MECHANICS

Basic relations

Mean velocity: $\overline{v} = \frac{1}{\rho A_c}\int_{A_c}\rho v\, dA_c$

$$\tau_w = \mu\left.\frac{\partial v}{\partial y}\right|_{y=0} = C_f\frac{\rho\overline{v}^2}{2} \qquad C_f = {}^{f}\!/_{4} \qquad D_h = \frac{4A_c}{p}$$

C_f = Fanning friction factor
f = Darcy friction factor

D_h = Hydraulic diameter
p = Wetted perimeter

Internal flow in pipes

$$f = \frac{64}{Re} \qquad \frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Laminar
smooth & rough

Turbulent
relative roughness ε/D

Power of a pump / fan: $\mathbb{P} = \Delta P_L\,\dot{V}$ $\qquad h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{\overline{v}^2}{2g}$

Conservation laws

Momentum balance:

$$\boldsymbol{F}_{net} \equiv \sum \boldsymbol{F} = \sum_{out} \beta_i \dot{m}_i \boldsymbol{v}_i - \sum_{in} \beta_i \dot{m}_i \boldsymbol{v}_i$$

Vector form, with β as the momentum flux correction factor

Mass flow:

$$\dot{m} = \rho \dot{V} = \rho \bar{v} A_c$$

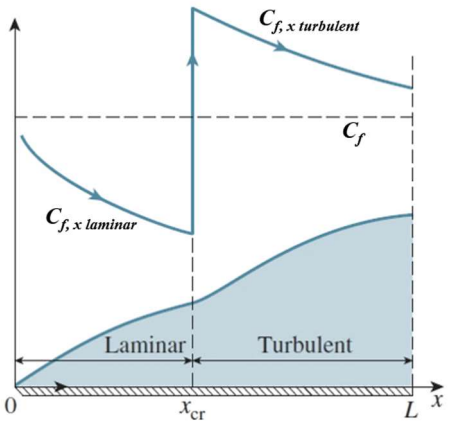
Mechanical energy balance:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{\bar{v}_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho g} + \alpha_2 \frac{\bar{v}_2^2}{2g} + z_2 + h_{turbine} + h_{loss}$$

With α as the kinetic energy correction factor

Parallel flow across plate

	Boundary layer	Friction factor	Condition
Laminar	$\delta = \frac{4.91x}{Re_x^{1/2}}$	$C_f = \frac{1.33}{Re_L^{1/2}}$	$Re_L < 5 \times 10^5$
Turbulent	$\delta = \frac{0.38x}{Re_x^{1/5}}$	$C_f = \frac{0.074}{Re_L^{1/5}}$	$5 \times 10^5 \leq Re_L < 10^7$
Combined result		$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L}$	$5 \times 10^5 \leq Re_L < 10^7$
Rough turbulent regime	$C_f = \left(1.89 - 1.62 \log_{10} \frac{\varepsilon}{L}\right)^{-2.5}$		$10^2 < \frac{L}{\varepsilon} < 10^6$



Flow inside packed bed

Ergun equation:

$$h_L = \frac{3 f_F (1-\varepsilon) L v_o^2}{g \varepsilon^3 d_p}$$

Packed-bed Reynolds number:

$$Re = \frac{\rho v_0 d_p}{(1-\varepsilon)\mu}$$

$$f_F = \tfrac{1}{3}[(150 \,/\, Re) + 1.75]$$

$$\frac{(P_1 - P_2) d_p \varepsilon^3}{\rho v_o^2 L (1-\varepsilon)} = \left[\frac{150}{Re} + 1.75\right] + \frac{d_p \varepsilon^3 g (z_2 - z_1)}{v_o^2 L (1-\varepsilon)}$$

Navier-Stokes

Navier-Stokes and Continuity Equations for an *Incompressible Newtonian* Fluid

[see *Fundamental Geometry* (page 8) for coordinate system]

Cartesian coordinates → (x, y, z)

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z$$

Continuity
$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Cylindrical coordinates → (r, θ, z)

r-component
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$
$$= -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + \rho g_r$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right) + \rho g_\theta$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

Continuity

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r}+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial v_z}{\partial z}=0$$

Spherical coordinates $\rightarrow (r,\theta,\phi)$

r-component

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\phi}{r}\frac{\partial v_r}{\partial \phi}+\frac{v_\theta}{r\sin\phi}\frac{\partial v_r}{\partial \theta}-\frac{v_\phi^2+v_\theta^2}{r}\right) = -\frac{\partial P}{\partial r}+\mu\left(\nabla^2 v_r-\frac{2v_r}{r^2}+\frac{2}{r^2}\frac{\partial v_\phi}{\partial \phi}-\frac{2}{r^2}v_\phi\cot\phi-\frac{2}{r^2\sin\phi}\frac{\partial v_\theta}{\partial \theta}\right)+\rho g_r$$

θ-component

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\phi}{r}\frac{\partial v_\theta}{\partial \phi}+\frac{v_\theta}{r\sin\phi}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}-\frac{v_\theta v_\phi\cot\phi}{r}\right) = -\frac{1}{r\sin\phi}\frac{\partial P}{\partial \theta}+\mu\left(\nabla^2 v_\theta-\frac{v_\theta}{r^2\sin^2\phi}+\frac{2}{r^2\sin\phi}\frac{\partial v_r}{\partial \theta}+\frac{2\cos\phi}{r^2\sin^2\phi}\frac{\partial v_\phi}{\partial \theta}\right)+\rho g_\theta$$

ϕ-component

$$\rho\left(\frac{\partial v_\phi}{\partial t}+v_r\frac{\partial v_\phi}{\partial r}+\frac{v_\phi}{r}\frac{\partial v_\phi}{\partial \phi}+\frac{v_\theta}{r\sin\phi}\frac{\partial v_\phi}{\partial \theta}+\frac{v_r v_\phi}{r}-\frac{v_\theta^2\cot\phi}{r}\right) = -\frac{1}{r}\frac{\partial P}{\partial \phi}+\mu\left(\nabla^2 v_\phi+\frac{2}{r^2}\frac{\partial v_r}{\partial \phi}-\frac{v_\phi}{r^2\sin^2\phi}+\frac{2\cos\phi}{r^2\sin^2\phi}\frac{\partial v_\theta}{\partial \theta}\right)+\rho g_\phi$$

where:

$$\nabla^2=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin^2\phi}\frac{\partial^2}{\partial \theta^2}+\frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial}{\partial \phi}\right)$$

Continuity

$$\frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r}+\frac{1}{r\sin\phi}\frac{\partial v_\theta}{\partial \theta}+\frac{1}{r\sin\phi}\frac{\partial \left(v_\phi\sin\phi\right)}{\partial \phi}=0$$

HEAT TRANSPORT

Fundamental laws

Energy conservation with *no work* done:

$$Q = mc_v \Delta T$$

Closed system

$$\dot{Q} = \dot{m} c_p \Delta T$$

Open system

Heat transfer mechanisms:

$$\dot{Q}_{cond} = -kA \frac{dT}{dx}$$

Fourier's Law

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

Newton's Law

***Note**: all above equations assume constant properties*

Fundamental definition of heat transfer coefficient (*local*):

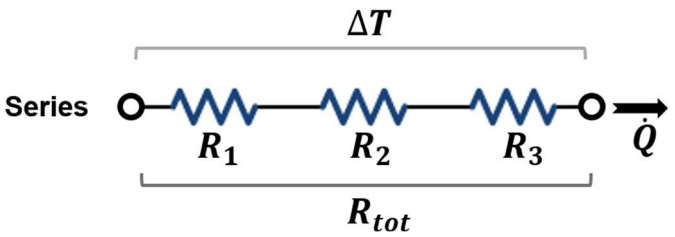
$$h = \frac{-k_{fluid} \, (\partial T / \partial y)_{y=0}}{T_s - T_\infty}$$

Stefan-Boltzmann's Law with net radiation, surrounded by infinite surface:

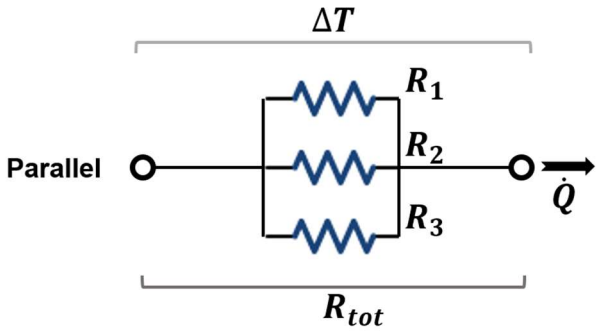
$$\dot{Q}_{rad} = \varepsilon \sigma A_s \left(T_s^4 - T_{surr}^4 \right)$$

Thermal circuit modelling

Applies to conduction, convection or radiation, with: $\dot{\boldsymbol{Q}} = \Delta \boldsymbol{T} / \boldsymbol{R}_{\mathbf{tot}}$



$$R_{tot} = R_1 + R_2 + R_3$$



$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Heat conduction equation

[see *Fundamental Geometry* (page 8) for coordinate system]

Cartesian coordinates → (x, y, z)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{gen} = \rho c_p \frac{\partial T}{\partial t}$$

Cylindrical coordinates → (r, θ, z)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_{gen} = \rho c_p \frac{\partial T}{\partial t}$$

Spherical coordinates → (r, θ, ϕ)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\phi}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(k\sin\phi\frac{\partial T}{\partial \phi}\right) + \dot{e}_{gen} = \rho c_p \frac{\partial T}{\partial t}$$

Finned surfaces

Standard fins of *constant* cross-section, A_c, and perimeter, p

Convective Tip

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL}$$

Adiabatic

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

Given Temperature

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\left(\frac{T_L - T_\infty}{T_b - T_\infty}\right)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

Infinite Fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx}$$

Fin parameter $m = \sqrt{\frac{hp}{kA_c}}$ *Corrected fin length* $L_c = L + \frac{A_c}{p}$

Lumped analysis

$$Bi = \frac{hL}{k_{solid}} \qquad \text{where } L = \frac{V}{A_s} \qquad Fo = \frac{\alpha t}{L^2}$$

With thermal diffusivity $\alpha = k/\rho c_p$

Note: the characteristic length used in Bi is normally based on L above – definitions may differ in charts!

External flow

$$h = \frac{1}{L}\int_0^L h_x\, dx \qquad \text{with} \qquad Nu_x = \frac{h_x x}{k}$$

Relative boundary layer thickness: $\frac{\delta}{\delta_t} \approx Pr^{1/3}$

Note: symbol h is conventionally used instead of $\overline{h}$ for the average h.t.c.

Internal flow

Bulk temperature: $\overline{T} = \frac{1}{\dot{m}c_p}\int_{A_c} \rho v\, c_p T\, dA_c$

Note: Exit and inlet temperatures are based on the bulk-mean value, $\overline{T}$

$$T_e(x) = T_s - (T_s - T_i)e^{-hA_s(x)/\dot{m}c_p} \qquad for \;\; T_s = constant$$

Entry lengths for developing flow

$$\frac{L_{h,laminar}}{D} \approx 0.05\, Re \qquad \frac{L_{t,laminar}}{D} \approx 0.05\, Re\, Pr \qquad L_{h,turbulent} \approx L_{t,turbulent} \approx 10D$$

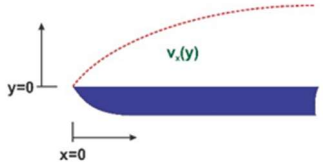
Boiling & Condensation

$$\dot{q}_{boil} = h(T_s - T_{sat}) = h\Delta T_{excess} \qquad \dot{m}_{evap} = \dot{Q}_{boil}/h_{fg}$$

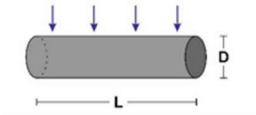
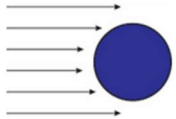
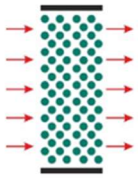
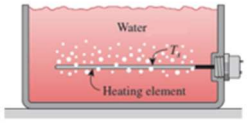
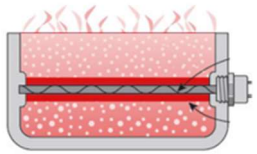
$$\dot{q}_{cond} = h(T_{sat} - T_s) \qquad \dot{m}_{cond} = \dot{Q}_{cond}/h^*_{fg}$$

Modified latent heat: $h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s) + c_{pv}(T_v - T_{sat})$

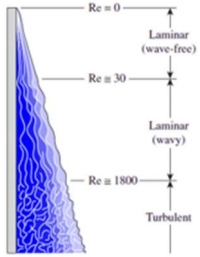
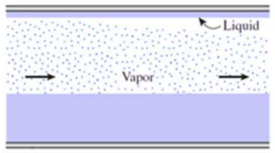
Correlations for convective *heat* transfer coefficient

Geometry		Correlation		Condition
	Laminar fully-developed flow	$Nu_{T_s=constant} = 3.66 \quad Nu_{\dot{q}=constant} = 4.36$		$Re < 2300$
	Laminar, thermally-developing flow ($T_s = constant$)	$Nu = 3.66 + \dfrac{0.065\,(D/L)\,Re\,Pr}{1 + 0.04[(D/L)\,Re\,Pr]^{2/3}}$		$Re < 2300$ $Pr > 5$
		$Nu = 1.86\left(\dfrac{Re\,Pr\,D}{L}\right)^{1/3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$		$Re < 2300$ $0.6 < Pr < 5$ $0.0044 < \mu_b/\mu_s < 9.75$
	Turbulent fully-developed flow (*smooth tubes*)	$Nu = 0.023\,Re^{0.8}Pr^{n}$	$n = 0.3$ (cooling) $n = 0.4$ (heating)	$Re > 10{,}000$ $0.7 < Pr < 160$
		$T_s = constant$: $Nu = 4.8 + 0.0156\,Re^{0.85}\,Pr_s^{0.93}$ $\dot{q}_s = constant$: $Nu = 6.3 + 0.0167\,Re^{0.85}\,Pr_s^{0.93}$		$10^4 < Re < 10^6$ $0.004 < Pr < 0.01$
	Parallel flow over plate (*local* Nu_x)	$Nu_x = \dfrac{h_x x}{k} = 0.332\,Re_x^{0.5}\,Pr^{1/3}$		$Re_x < 5 \times 10^5$ $Pr > 0.6$
		$Nu_x = \dfrac{h_x x}{k} = 0.0296\,Re_x^{0.8}\,Pr^{1/3}$		$5 \times 10^5 < Re_x < 10^7$ $0.6 < Pr < 60$
	Parallel flow over plate (*averaged* Nu)	$Nu = \dfrac{hL}{k} = 0.664\,Re_L^{0.5}\,Pr^{1/3}$		$Re_L < 5 \times 10^5$ $Pr > 0.6$
		$Nu = \dfrac{hL}{k} = 0.037\,Re_L^{0.8}\,Pr^{1/3}$		$5 \times 10^5 < Re_L < 10^7$ $0.6 < Pr < 60$
		$Nu = \dfrac{hL}{k} = \left(0.037\,Re_L^{0.8} - 871\right)Pr^{1/3}$	(combined) incl. laminar section	$5 \times 10^5 < Re_L < 10^7$ $0.6 < Pr < 60$
	Parallel flow over plate (*local* Nu_x) ($\dot{q} = constant$)	$Nu_x = \dfrac{h_x x}{k} = 0.453\,Re_x^{0.5}\,Pr^{1/3}$		$Re_x < 5 \times 10^5$ $Pr > 0.6$
		$Nu_x = \dfrac{h_x x}{k} = 0.0308\,Re_x^{0.8}\,Pr^{1/3}$		$5 \times 10^5 < Re_x < 10^7$ $0.6 < Pr < 60$

Correlations for convective *heat* transfer coefficient

Geometry		Correlation		Condition
	Uniform flow over smooth cylinder	$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\ Re^{0.5}\ Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5}$		$Re\ Pr > 0.2$
	Uniform flow over smooth sphere	$Nu = \frac{hD}{k} = 2 + \left[0.4\ Re^{0.5} + 0.06\ Re^{2/3}\right] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$		$3.5 < Re < 8 \times 10^4$ $0.7 < Pr < 380$ $1.0 < \mu_\infty/\mu_s < 3.2$
	Flow through tube banks (*in-line & staggered*)	$Nu = \frac{hD}{k} = C Re_D^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$	C, m, n from tables	$Re_D < 2 \times 10^6$ $0.7 < Pr < 500$
	Nucleate boiling	$\dot{q}_{nucleate} = \mu_l h_{fg} \left[\frac{(\rho_l - \rho_v)g}{\sigma}\right]^{0.5} \left[\frac{c_{pl}(T_s - T_{sat})}{C_{sf} h_{fg} Pr_l^n}\right]^3$	C_{sf}, n from tables	$5°C < \Delta T_{excess} < 30°C$
		$\dot{q}_{max} = C_{cr}\ h_{fg}\left[\sigma g \rho_v^2 (\rho_l - \rho_v)\right]^{1/4}$	C_{cr} from tables	
	Film boiling	$\dot{q}_{film} = C_{film} \left[\frac{g k_v^3 \rho_v (\rho_l - \rho_v) h_{fg} + 0.4 c_{pv}(T_s - T_{sat})}{\mu_v D (T_s - T_{sat})}\right]^{1/4} (T_s - T_{sat})$	$C_{film} = 0.62$ (h. cylinders) $C_{film} = 0.67$ (spheres)	$\Delta T_{excess} > 120°C$
		$\dot{q}_{min} = 0.09 \rho_v\ h_{fg} \left[\frac{\sigma g (\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{1/4}$		

Correlations for convective *heat* transfer coefficient

Geometry		Correlation		Condition
	Film condensation on vertical plate	$h_{vert} = 0.943 \left[\frac{g\rho_l(\rho_l - \rho_v)\, h^*_{fg} k_l^3}{\mu_l (T_{sat} - T_s) L}\right]^{1/4}$		$Re < 30$
		$h_{vert,wavy} = \frac{Re\, k_l}{1.08\, Re^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$		$30 < Re < 1800$
		$h_{vert,turb} = \frac{Re\, k_l}{8750 + 58\, Pr^{-0.5}\left(Re^{0.75} - 253\right)}\left(\frac{g}{\nu_l^2}\right)^{1/3}$		$Re > 1800$
	Film condensation on horizontal tube	$h_{horiz} = C_c \left[\frac{g\rho_l(\rho_l - \rho_v)\, h^*_{fg} k_l^3}{\mu_l (T_{sat} - T_s) L}\right]^{1/4}$	$C_c = 0.729$ (h. cylinders) $C_c = 0.815$ (spheres)	$Re < 30$
	Film condensation inside horizontal tube	$h_{internal} = 0.555 \left[\frac{g\rho_l(\rho_l - \rho_v)\, k_l^3}{\mu_l (T_{sat} - T_s) D}\left(h_{fg} + \frac{3}{8} c_{pl}(T_{sat} - T_s)\right)\right]^{1/4}$		$Re_{vapor} < 35{,}000$
	Dropwise condensation	$h_{dropwise} = 51{,}104 + 2044 T_{sat}$		$22\ ^\circ C < T_{sat} < 100\ ^\circ C$
		$h_{dropwise} = 255{,}510$		$T_{sat} > 100\ ^\circ C$

Heat exchangers

Log-mean temperature difference:

$$\Delta T_{LM} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

where $\Delta T_e = T_s - T_e$ and $\Delta T_i = T_s - T_i$

Overall heat transfer coefficient:

$$\frac{1}{U A_s} \equiv \frac{1}{U_i A_i} \equiv \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of the inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

MASS TRANSPORT

Fundamental laws

Mass transfer mechanisms:

$$J_{A,x} = -\mathfrak{D}_{AB}\frac{dC_A}{dx}$$

Fickian diffusion

$$N_A = k_c\left(C_{A,s} - C_{A,\infty}\right)$$

Convection

Fundamental definition of mass transfer coefficient (*local*):

$$k_c = \frac{-\mathfrak{D}_{AB}\,(\partial C_A/\partial y)_{y=0}}{C_{A,s} - C_{A,\infty}}$$

Note: The following framework is based on the assumption of *binary* mixtures. For multicomponent systems refer to the literature.

Mass Diffusivity

$$\mathfrak{D}_{AB} = \frac{0.001858\, T^{3/2}\left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}}{P\sigma_{AB}^2\Omega_D(T)}$$

Hirschfelder's equation for binary **gas** phase diffusivity ♦

$$\frac{\mathfrak{D}_{AB}\mu_B}{T} = \frac{7.4\times10^{-8}(\Phi_B M_B)^{1/2}}{V_A^{0.6}}$$

Wilke-Chang correlation for binary **liquid** phase diffusivity ♠

$$\mathfrak{D}_{KA} = 4850\, d_{pore}\sqrt{\frac{T}{M_A}}$$

Knudsen diffusivity in **porous** solid

$$\mathfrak{D}_{1-mixture} = \frac{1}{\frac{y'_2}{\mathfrak{D}_{12}}+\frac{y'_3}{\mathfrak{D}_{13}}+\cdots+\frac{y'_n}{\mathfrak{D}_{1n}}}$$

Diffusivity of minor species in a gas **mixture**

$$\text{with}\quad y'_2 = \frac{y_2}{y_2+y_3+\cdots+y_n}$$

♦ Parameter σ_{AB} and $\Omega_D(T)$ is the *collision diameter* and *collision integral* respectively, whose values are given elsewhere.

♠ Parameter V_A is the *molar volume* of solute A at its normal boiling point, and Φ_B is the *association parameter* for solvent B - whose values are given elsewhere.

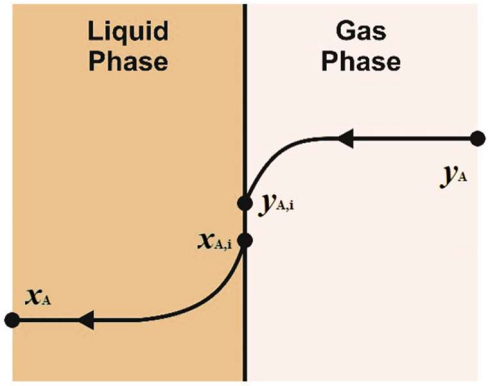
Mass transfer between fluid phases

	Concentration units	Flux equation	Units of k (in *SI*)
Liquid Film	Mole concentration	$N_A = k_{cL}\left(C_{AL,i} - C_{AL}\right)$	*m/s*
	Mole fraction	$N_A = k_x\left(x_{A,i} - x_A\right)$	*mol/(m².s)*
Gas Film	Partial pressure	$N_A = k_p\left(P_A - P_{A,i}\right)$	*mol/(m².Pa.s)*
	Mole fraction	$N_A = k_y\left(y_A - y_{A,i}\right)$	*mol/(m².s)*
	Mole concentration	$N_A = k_{cG}\left(C_{AG} - C_{AG,i}\right)$	*m/s*

Liquid phase:

$$k_{cL} = \frac{k_x}{C_L}$$

Where C_L is the total molar concentration in the liquid phase.



Gas phase (*ideal*):

$$k_p = \frac{k_y}{P} = \frac{k_{cG}}{RT}$$

Where P is the total pressure.

Overall mass transfer coefficient for linear equilibrium (Henry's law)

Units	Equilibrium relation	Liquid phase	Gas phase
Concentrations and pressure	$P_A = H\, C_A$	$\frac{1}{K_L} = \frac{1}{H\, k_p} + \frac{1}{k_{cL}}$	$\frac{1}{K_G} = \frac{1}{k_p} + \frac{H}{k_{cL}}$
Mole fractions	$y_A = m\, x_A$	$\frac{1}{K_x} = \frac{1}{m\, k_y} + \frac{1}{k_x}$	$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}$

External flow

$$k_c = \frac{1}{L}\int_0^L k_{c,x}\, dx \qquad \text{with} \qquad Sh_x = \frac{k_{c,x}\, x}{\mathfrak{D}_{AB}}$$

Relative boundary layer thickness: $\frac{\delta}{\delta_c} \approx Sc^{1/3}$

Note: symbol k_c is conventionally used instead of $\overline{k}_c$ for the average m.t.c.

Internal flow

Mean (Bulk) mass fraction: $$\overline{w} = \frac{1}{\dot{m}}\int_{A_c} \rho v\, w \;\; dA_c$$

Entry length for developing flow: $$\frac{L_{c,laminar}}{D} \approx 0.05\; Re\; Sc$$

Mass transport equation

$$\frac{\partial C_A}{\partial t} + \nabla \bullet \boldsymbol{N_A} = r_A$$

Gradient of a scalar quantity (y_A, C_A, …):

[see *Fundamental Geometry* (page 8) for coordinate system]

Cartesian coordinates → (x, y, z)

$$\nabla y_A = \frac{\partial y_A}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial y_A}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial y_A}{\partial z}\hat{\boldsymbol{z}}$$

Cylindrical coordinates → (r, θ, z)

$$\nabla y_A = \frac{\partial y_A}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial y_A}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{\partial y_A}{\partial z}\hat{\boldsymbol{z}}$$

Spherical coordinates → (r, θ, ϕ)

$$\nabla y_A = \frac{\partial y_A}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r\sin\phi}\frac{\partial y_A}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r}\frac{\partial y_A}{\partial \phi}\hat{\boldsymbol{\phi}}$$

Divergence of a vector quantity (N_A, v, …):

Cartesian coordinates → (x, y, z)

$$\nabla \bullet \boldsymbol{N_A} = \frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} + \frac{\partial N_{A,z}}{\partial z}$$

Cylindrical coordinates → (r, θ, z)

$$\nabla \bullet \boldsymbol{N_A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rN_{A,r}\right) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}$$

Spherical coordinates → (r, θ, ϕ)

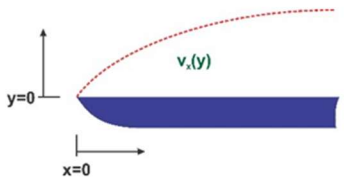
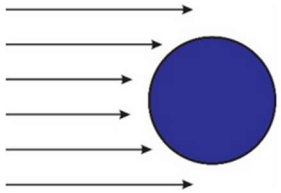
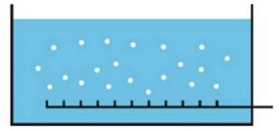
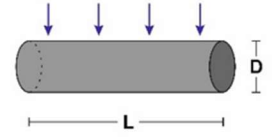
$$\nabla \bullet \boldsymbol{N_A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 N_{A,r}\right) + \frac{1}{r\sin\phi}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{1}{r\sin\phi}\frac{\partial}{\partial \phi}\left(N_{A,\phi}\sin\phi\right)$$

Equivalent forms of *molar* flux for a binary system

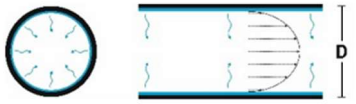
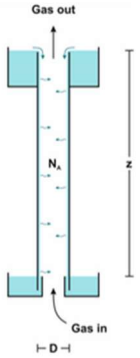
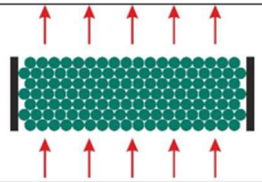
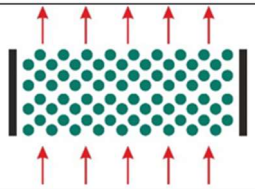
	Restriction		Diffusive	Diffusive + Convective
	Const. T	*Const. P*		
Liquid (*ideal solution*)	✓		$\boldsymbol{J_A} = -C_L\mathfrak{D}_{AB}\nabla x_A$	$\boldsymbol{N_A} = -C_L\mathfrak{D}_{AB}\nabla x_A + x_A(\boldsymbol{N_A} + \boldsymbol{N_B})$
Gas (*ideal*)	✓	✓	$\boldsymbol{J_A} = -C_G\mathfrak{D}_{AB}\nabla y_A$	$\boldsymbol{N_A} = -C_G\mathfrak{D}_{AB}\nabla y_A + y_A(\boldsymbol{N_A} + \boldsymbol{N_B})$
	✓		$\boldsymbol{J_A} = -(\mathfrak{D}_{AB}/RT)\nabla P_A$	$\boldsymbol{N_A} = -(\mathfrak{D}_{AB}/RT)\nabla P_A + y_A(\boldsymbol{N_A} + \boldsymbol{N_B})$
	✓	✓	$\boldsymbol{J_A} = -\mathfrak{D}_{AB}\nabla C_A$	$\boldsymbol{N_A} = -\mathfrak{D}_{AB}\nabla C_A + y_A(\boldsymbol{N_A} + \boldsymbol{N_B})$

Where mass fluxes are related via: $\boldsymbol{j_A} = M_A\,\boldsymbol{J_A}$ and $\boldsymbol{n_A} = M_A\,\boldsymbol{N_A}$

Correlations for convective *mass* transfer coefficient

Geometry		Fluid	Correlation	Condition
	Parallel flow over plate (*local* Sh_x)	Gas or liquid	$Sh_x = \frac{k_{c,x}x}{\mathfrak{D}_{AB}} = 0.332\ Re_x^{0.5} Sc^{1/3}$	$Re_x < 2\times10^5$ $Sc > 0.6$
		Gas or liquid	$Sh_x = \frac{k_{c,x}x}{\mathfrak{D}_{AB}} = 0.0292\ Re_x^{0.8} Sc^{1/3}$	$Re_x > 2\times10^5$ $Sc > 0.6$
	Parallel flow over plate (*averaged Sh*)	Gas or liquid	$Sh = \frac{k_c L}{\mathfrak{D}_{AB}} = 0.664\ Re_L^{0.5} Sc^{1/3}$	$Re_L < 2\times10^5$ $Sc > 0.6$
		Gas or liquid	$Sh = 0.664\ Re_t^{0.5} Sc^{1/3} + 0.0365\ Sc^{1/3}\left(Re_t^{0.8} - Re_L^{0.8}\right)$	$2\times10^5 < Re < 3\times10^6$ $Sc > 0.6$
		Gas or liquid	$Sh = \frac{k_c L}{\mathfrak{D}_{AB}} = 0.0365\ Re_L^{0.8} Sc^{1/3}$	$Re_L > 3\times10^6$ $Sc > 0.6$
	Uniform flow over smooth sphere **(Forced Convection)**	Liquid	$Sh = \frac{k_c D}{\mathfrak{D}_{AB}} = \left(4 + 1.21\ Pe_{AB}^{2/3}\right)^{0.5}$	$Pe_{AB} < 10{,}000$ $Re \geq 0.4\ Gr^{1/2} Sc^{-1/6}$
			$Sh = \frac{k_c D}{\mathfrak{D}_{AB}} = 1.01\ Pe_{AB}^{1/3}$	$Pe_{AB} > 10{,}000$ $Re \geq 0.4\ Gr^{1/2} Sc^{-1/6}$
		Gas	$Sh = \frac{k_c D}{\mathfrak{D}_{AB}} = 2 + 0.552\ Re^{0.5} Sc^{1/3}$	$2 < Re < 12{,}000$ $0.6 < Sc < 1.85$ $Re \geq 0.4\ Gr^{1/2} Sc^{-1/6}$
	Buoyant flow from smooth sphere **(Natural Convection)**	Gas or liquid	$Sh = Sh^* + 0.347\left(Re\ Sc^{0.5}\right)^{0.62}$ $Sh^* = 2 + 0.569(Gr\ Sc)^{0.25}$	$Gr\ Sc \leq 10^8$ $2 < Re < 30{,}000$ $0.6 < Sc < 3200$
		Gas or liquid	$Sh = Sh^{**} + 0.347\left(Re\ Sc^{0.5}\right)^{0.62}$ $Sh^{**} = 2 + 0.0254(Gr\ Sc)^{0.5}(Sc)^{0.244}$	$Gr\ Sc > 10^8$ $2 < Re < 30{,}000$ $0.6 < Sc < 3200$
	Sparger with spherical bubbles	Gas	$Sh = 0.31\ Gr^{1/3} Sc^{1/3}$	$d_p < 2.5\ mm$
		Gas	$Sh = 0.42\ Gr^{1/3} Sc^{0.5}$	$d_p \geq 2.5\ mm$
	Uniform flow over smooth cylinder	Gas	$\frac{k_p P (Sc)^{0.56}}{G_M} = \frac{k_c (Sc)^{0.56}}{v_\infty} = 0.281 (Re)^{-0.4}$	$400 < Re < 25{,}000$ $0.6 < Sc < 2.6$

Correlations for convective *mass* transfer coefficient

Geometry		Fluid	Correlation		Condition
	Flow inside tube	Gas	$\frac{k_c D}{\mathfrak{D}_{AB}} \frac{P_{B,LM}}{P} = 0.023\ Re^{0.83} Sc^{0.44}$		$2000 < Re < 35{,}000$ $0.6 < Sc < 2.5$
		Liquid	$Sh = \frac{k_c D}{\mathfrak{D}_{AB}} = 0.023\ Re^{0.83} Sc^{1/3}$		$2000 < Re < 70{,}000$ $1000 < Sc < 2260$
		Gas or liquid	$Sh = 1.86 \left(\frac{v_\infty D^2}{L\, \mathfrak{D}_{AB}} \right)^{1/3} = 1.86 \left(\frac{D}{L} Re\ Sc \right)^{1/3}$		$Re < 2000, \quad Re\ Sc \left(\frac{D}{L} \right) > 10$
	Mass transfer in wetted wall column	Liquid Film	$Sh = \frac{k_c z}{\mathfrak{D}_{AB}} = 0.433 (Sc)^{1/2} \left(\frac{\rho_l^2 g z^3}{\mu_l^2} \right)^{1/6} (Re_z)^{0.4}$		$Re_z = \frac{4\dot{m}_l}{\pi D \mu_l}$
	Mass transfer for flow inside *packed* bed of spheres	Liquid	$\varepsilon j_D = \frac{1.09}{Re}$	$j_D = \frac{k_c}{v_0} Sc^{2/3}$	$165 < Sc < 70{,}600$ $0.35 < \varepsilon < 0.75$
		Liquid	$\varepsilon j_D = \frac{0.25}{Re^{0.31}}$		$55 < Re < 1500$ $165 < Sc < 10{,}690$
		Gas	$\varepsilon j_D = \frac{2.06}{Re^{0.575}}$		$90 < Re < 4000$
	Mass transfer for flow inside *fluidized* bed of spheres	Gas or liquid	$\varepsilon j_D = 0.01 + \frac{0.863}{Re^{0.58} - 0.483}$	$Re = \frac{\rho d_p v_0}{\mu}$	$90 < Re < 4000$

SEPARATION PROCESSES

Flashing & Distillation

Rachford-Rice

$$\sum_{i=1}^{C}(y_i - x_i) = \sum_{i=1}^{C}\frac{(K_i - 1)z_i}{1 + (K_i - 1)\dfrac{V}{F}} = 0$$

Fenske

$$N_{min} = \frac{\ln\left[\dfrac{(FR_{LK})_{dist}(FR_{HK})_{bot}}{\left[1 - (FR_{LK})_{dist}\right]\left[1 - (FR_{HK})_{bot}\right]}\right]}{\ln \alpha_{avg}}$$

$$\alpha_{avg} = \sqrt[3]{\alpha_{feed}\alpha_{dist}\alpha_{bot}}$$

Modified Fenske

$$N_{F,min} = \frac{\ln\left[\dfrac{\left(\dfrac{x_{LK}}{x_{HK}}\right)_{dist}}{\left(\dfrac{z_{LK}}{z_{HK}}\right)}\right]}{\ln \alpha_{avg}}$$

$$\frac{N_{F,min}}{N_{min}} = \frac{N_F}{N}$$

First Underwood

$$F(1 - q) = V_{min} - \overline{V}_{min} = \sum_{i=1}^{C}\frac{\alpha_{i-ref}Fz_i}{\alpha_{i-ref} - \phi}$$

Second Underwood

$$V_{min} = \sum_{i=1}^{C}\frac{\alpha_{i-ref}Dx_{i,dist}}{\alpha_{i-ref} - \phi}$$

Kirkbride correlation

$$\log_{10}\left(\frac{N_F - 1}{N - N_F}\right) = 0.26\log_{10}\left[\frac{B}{D}\left(\frac{z_{HK}}{z_{LK}}\right)\left(\frac{x_{LK,bot}}{x_{HK,dist}}\right)^2\right]$$

O'Connell correlation

$$E_o = 0.52782 - 0.27511\log_{10}(\alpha\mu) + 0.044923\left[\log_{10}(\alpha\mu)\right]^2$$

Absorption & Stripping

Kremser

$$\frac{y_B - y_B^*}{y_T - y_T^*} = A^N$$

with the absorption factor, $A \equiv \dfrac{L}{mG}$

Flooding velocity

$$u_{flood} = C_{sb,f}\left(\frac{\sigma}{20}\right)^{0.2}\sqrt{\frac{\rho_L - \rho_V}{\rho_V}}$$

PHYSICAL PROPERTIES OF AIR & WATER

Air

Average molar mass	$\overline{M}_R = 29$ g/mol
Specific gas constant	$\hat{R} = 287$ J/(kg K)
Specific heat capacities at 298 K	$c_p = 1005$ J/(kg K) $c_v = 718$ J/(kg K) $\gamma = 1.40$

Composition	**Mole %**	**Mass %**
O_2	21.0	23.1
N_2	78.1	75.6
Ar	0.9	1.3

Viscosities and Thermal Conductivity at absolute pressure of 1 bar

T	**0**	**20**	**40**	**60**	**80**	**100**		**°C**
μ	1.71	1.81	1.90	2.00	2.09	2.18	$\times 10^{-5}$	Pa s
ν	1.32	1.50	1.69	1.88	2.09	2.30	$\times 10^{-5}$	m^2/s
k	0.024	0.025	0.027	0.028	0.029	0.031		W/(m K)

Water

Specific heat capacity at 298 K	$c_p = 4187$ J/(kg K)
Surface tension with air at 298 K	$\sigma = 0.073$ N/m

Viscosities, Thermal Conductivity and Vapour Pressure

T	**0**	**20**	**40**	**60**	**80**	**100**		**°C**
μ	1.79	1.01	0.656	0.469	0.357	0.284	$\times 10^{-3}$	Pa s
ν	1.79	1.01	0.661	0.477	0.367	0.296	$\times 10^{-6}$	m^2/s
k	0.57	0.60	0.63	0.65	0.67	0.68		W/(m K)
P^{sat}	0.61	2.34	7.38	19.9	47.4	101.3		kPa

TRANSPORT PROPERTIES

The below values are correct for a pressure of 1 atm = 1.01325 bar, but may be used with sufficient accuracy at other reasonable pressures.

Air

T	c_p	k	μ / 10^{-6}	$Pr = \mu c_p/k$
°C	kJ/(kg K)	W/(m K)	Pa s	
-100	1.01	0.016	12	0.75
0	1.01	0.024	17	0.72
100	1.02	0.032	22	0.70
200	1.03	0.039	26	0.69
300	1.05	0.045	30	0.69
400	1.07	0.051	33	0.70
500	1.10	0.056	36	0.70
600	1.12	0.061	39	0.71
700	1.14	0.066	42	0.72
800	1.16	0.071	44	0.73

This table may be used with reasonable accuracy for values of c_p, k, μ and Pr of N_2, O_2 and CO.

Steam

T	c_p	k	μ / 10^{-6}	$Pr = \mu c_p/k$
°C	kJ/(kg K)	W/(m K)	Pa s	
100	2.028	0.0245	12.1	0.986
200	1.979	0.0331	16.2	0.968
300	2.010	0.0434	20.4	0.946
400	2.067	0.0548	24.6	0.928
500	2.132	0.0673	28.8	0.912
600	2.201	0.0805	32.9	0.898
700	2.268	0.0942	36.8	0.887
800	2.332	0.1080	40.6	0.876

THERMODYNAMIC DATA FOR WATER & STEAM

Source of data

The following data tables have been sourced from IAPWS – the international association of the properties of water and steam website: www.iapws.org

The arbitrary datum chosen is that saturated liquid water at the triple point has internal energy $U = 0$ and entropy $S = 0$

Triple point data

Temperature = 273.16 K (0.01 ℃)

Pressure = 0.00611 bar

Phase	Specific volume	Specific enthalpy	Specific entropy
	m^3/kg	kJ/kg	kJ/(kg K)
Ice	0.0010905	−333.5	−1.221
Water	0.0010002	0.0062	**0.0**
Steam	206	2500.9	9.156

Critical point data

Temperature = 647.1 K (374 ℃)

Pressure = 220.64 bar

Density = 322 kg/m^3

SATURATED STEAM TABLES

Temperatures from the triple point → 100 °C

Temp.	Pressure	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)			S (kJ/kg K)		Temp.
(°C)	(bar)	water	steam	water	steam	water	evap'n	steam	water	steam	(°C)
0.01	**0.00611**	**0.001000**	**206.005**	**zero**	**2375.0**	**0.0**	**2500.9**	**2500.9**	**zero**	**9.156**	**0.01**
2	0.00706	0.001000	179.776	8.4	2377.6	8.4	2496.2	2504.6	0.031	9.103	2
4	0.00814	0.001000	157.135	16.8	2380.4	16.8	2491.4	2508.2	0.061	9.051	4
6	0.00935	0.001000	137.652	25.2	2383.1	25.2	2486.7	2511.9	0.091	8.999	6
8	0.01073	0.001000	120.846	33.6	2385.9	33.6	2481.9	2515.6	0.121	8.949	8
10	0.01228	0.001000	106.319	42.0	2388.6	42.0	2477.2	2519.2	0.151	8.900	10
12	0.01403	0.001001	93.732	50.4	2391.4	50.4	2472.5	2522.9	0.181	8.851	12
14	0.01599	0.001001	82.804	58.8	2394.1	58.8	2467.7	2526.5	0.210	8.804	14
16	0.01819	0.001001	73.295	67.2	2396.9	67.2	2463.0	2530.2	0.239	8.757	16
18	0.02065	0.001001	65.005	75.5	2399.6	75.5	2458.3	2533.8	0.268	8.711	18
20	0.02339	0.001002	57.762	83.9	2402.3	83.9	2453.5	2537.4	0.296	8.666	20
22	0.02645	0.001002	51.422	92.3	2405.0	92.3	2448.8	2541.1	0.325	8.622	22
24	0.02986	0.001003	45.861	100.6	2407.8	100.6	2444.1	2544.7	0.353	8.578	24
25	**0.03170**	**0.001003**	**43.340**	**104.8**	**2409.1**	**104.8**	**2441.7**	**2546.5**	**0.367**	**8.557**	**25**
26	0.03364	0.001003	40.975	109.0	2410.5	109.0	2439.3	2548.3	0.381	8.535	26
28	0.03783	0.001004	36.673	117.4	2413.2	117.4	2434.6	2551.9	0.409	8.493	28
30	0.04247	0.001004	32.879	125.7	2415.9	125.7	2429.8	2555.5	0.437	8.452	30
32	0.04760	0.001005	29.527	134.1	2418.6	134.1	2425.1	2559.2	0.464	8.411	32
34	0.05325	0.001006	26.560	142.4	2421.3	142.4	2420.3	2562.8	0.492	8.371	34
36	0.05948	0.001006	23.929	150.8	2424.0	150.8	2415.5	2566.3	0.519	8.332	36
38	0.06633	0.001007	21.593	159.2	2426.7	159.2	2410.8	2569.9	0.546	8.294	38
40	0.07385	0.001008	19.515	167.5	2429.4	167.5	2406.0	2573.5	0.572	8.256	40
42	0.08210	0.001009	17.663	175.9	2432.1	175.9	2401.2	2577.1	0.599	8.218	42
44	0.09113	0.001010	16.010	184.2	2434.7	184.2	2396.4	2580.6	0.625	8.181	44
46	0.10100	0.001010	14.534	192.6	2437.4	192.6	2391.6	2584.2	0.652	8.145	46
48	0.11178	0.001011	13.212	201.0	2440.1	201.0	2386.8	2587.8	0.678	8.110	48
50	0.12352	0.001012	12.026	209.3	2442.7	209.3	2382.0	2591.3	0.704	8.075	50
52	0.13632	0.001013	10.962	217.7	2445.4	217.7	2377.1	2594.8	0.730	8.040	52
54	0.15023	0.001014	10.006	226.0	2448.0	226.1	2372.3	2598.3	0.755	8.007	54
56	0.16534	0.001015	9.145	234.4	2450.7	234.4	2367.4	2601.8	0.781	7.973	56
58	0.18172	0.001016	8.368	242.8	2453.3	242.8	2362.5	2605.3	0.806	7.940	58
60	0.19947	0.001017	7.667	251.1	2455.9	251.2	2357.7	2608.8	0.831	7.908	60
62	0.21868	0.001018	7.033	259.5	2458.5	259.5	2352.8	2612.3	0.856	7.876	62
64	0.23944	0.001019	6.460	267.9	2461.1	267.9	2347.9	2615.8	0.881	7.845	64
66	0.26184	0.001020	5.940	276.3	2463.7	276.3	2342.9	2619.2	0.906	7.814	66
68	0.28600	0.001022	5.468	284.6	2466.3	284.7	2338.0	2622.7	0.931	7.784	68
70	0.31202	0.001023	5.040	293.0	2468.8	293.1	2333.0	2626.1	0.955	7.754	70
72	0.34002	0.001024	4.650	301.4	2471.4	301.4	2328.1	2629.5	0.979	7.725	72
74	0.37010	0.001025	4.295	309.8	2474.0	309.8	2323.1	2632.9	1.004	7.696	74
76	0.40240	0.001026	3.971	318.2	2476.5	318.2	2318.1	2636.3	1.028	7.667	76
78	0.43704	0.001028	3.675	326.6	2479.0	326.6	2313.1	2639.7	1.052	7.639	78
80	0.47416	0.001029	3.405	335.0	2481.5	335.0	2308.0	2643.0	1.076	7.611	80
82	0.51388	0.001030	3.158	343.3	2484.1	343.4	2303.0	2646.4	1.099	7.584	82
84	0.55636	0.001032	2.932	351.7	2486.5	351.8	2297.9	2649.7	1.123	7.557	84
86	0.60174	0.001033	2.725	360.1	2489.0	360.2	2292.8	2653.0	1.146	7.530	86
88	0.65018	0.001035	2.534	368.6	2491.5	368.6	2287.6	2656.3	1.170	7.504	88
90	0.70183	0.001036	2.359	377.0	2494.0	377.0	2282.5	2659.5	1.193	7.478	90
92	0.75685	0.001037	2.198	385.4	2496.4	385.4	2277.3	2662.8	1.216	7.453	92
94	0.81542	0.001039	2.050	393.8	2498.8	393.9	2272.1	2666.0	1.239	7.428	94
96	0.87771	0.001040	1.914	402.2	2501.2	402.3	2266.9	2669.2	1.262	7.403	96
98	0.94390	0.001042	1.788	410.6	2503.6	410.7	2261.7	2672.4	1.285	7.378	98
100	1.01418	0.001043	1.672	419.1	2506.0	419.2	2256.4	2675.6	1.307	7.354	100

SATURATED STEAM TABLES

Temperatures from 100 °C → critical point

Temp.	Pressure	V (m^3/kg)		U (kJ/kg)		H (kJ/kg)			S (kJ/kg K)		Temp.
(°C)	(bar)	water	steam	water	steam	water	evap'n	steam	water	steam	(°C)
100	1.014	0.001043	1.67196	419.1	2506.0	419.2	2256.4	2675.6	1.307	7.354	100
105	1.209	0.001047	1.41856	440.1	2511.9	440.3	2243.1	2683.4	1.363	7.295	105
110	1.434	0.001052	1.20945	461.3	2517.7	461.4	2229.6	2691.1	1.419	7.238	110
115	1.692	0.001056	1.03598	482.4	2523.3	482.6	2216.0	2698.6	1.474	7.183	115
120	1.987	0.001060	0.89133	503.6	2528.8	503.8	2202.1	2705.9	1.528	7.129	120
125	2.322	0.001065	0.77012	524.8	2534.3	525.1	2188.0	2713.1	1.582	7.077	125
130	2.703	0.001070	0.66808	546.1	2539.5	546.4	2173.7	2720.1	1.635	7.026	130
135	3.132	0.001075	0.58179	567.4	2544.6	567.7	2159.1	2726.9	1.687	6.977	135
140	3.615	0.001080	0.50850	588.8	2549.6	589.2	2144.3	2733.4	1.739	6.929	140
145	4.157	0.001085	0.44600	610.2	2554.4	610.6	2129.2	2739.8	1.791	6.883	145
150	4.762	0.001091	0.39248	631.7	2559.0	632.2	2113.7	2745.9	1.842	6.837	150
155	5.435	0.001096	0.34648	653.2	2563.5	653.8	2098.0	2751.8	1.892	6.793	155
160	6.182	0.001102	0.30680	674.8	2567.8	675.5	2082.0	2757.4	1.943	6.749	160
165	7.009	0.001108	0.27244	696.5	2571.8	697.2	2065.6	2762.8	1.992	6.707	165
170	7.922	0.001114	0.24260	718.2	2575.7	719.1	2048.8	2767.9	2.042	6.665	170
175	8.926	0.001121	0.21659	740.0	2579.4	741.0	2031.7	2772.7	2.091	6.624	175
180	10.028	0.001127	0.19384	761.9	2582.8	763.1	2014.2	2777.2	2.139	6.584	180
185	11.235	0.001134	0.17390	783.9	2586.0	785.2	1996.2	2781.4	2.188	6.545	185
190	12.552	0.001141	0.15636	806.0	2589.0	807.4	1977.8	2785.3	2.235	6.506	190
195	13.988	0.001149	0.14089	828.2	2591.7	829.8	1959.0	2788.8	2.283	6.468	195
200	15.549	0.001157	0.12721	850.5	2594.2	852.3	1939.7	2792.0	2.331	6.430	200
205	17.243	0.001164	0.11508	872.9	2596.4	874.9	1919.9	2794.8	2.378	6.393	205
210	19.077	0.001173	0.10429	895.4	2598.3	897.6	1899.6	2797.3	2.424	6.356	210
215	21.059	0.001181	0.09468	918.0	2599.9	920.5	1878.8	2799.3	2.471	6.320	215
220	23.196	0.001190	0.08609	940.8	2601.2	943.6	1857.4	2800.9	2.518	6.284	220
225	25.497	0.001199	0.07841	963.7	2602.2	966.8	1835.3	2802.2	2.564	6.248	225
230	27.971	0.001209	0.07151	986.8	2602.9	990.2	1812.7	2802.9	2.610	6.213	230
235	30.626	0.001219	0.06530	1010.0	2603.2	1013.8	1789.4	2803.2	2.656	6.178	235
240	33.470	0.001229	0.05971	1033.4	2603.1	1037.6	1765.4	2803.0	2.702	6.142	240
245	36.512	0.001240	0.05466	1057.0	2602.7	1061.5	1740.7	2802.2	2.748	6.107	245
250	39.762	0.001252	0.05009	1080.8	2601.8	1085.8	1715.2	2800.9	2.793	6.072	250
255	43.229	0.001263	0.04594	1104.8	2600.5	1110.2	1688.9	2799.1	2.839	6.037	255
260	46.923	0.001276	0.04218	1129.0	2598.7	1134.9	1661.7	2796.6	2.885	6.002	260
265	50.853	0.001289	0.03875	1153.4	2596.5	1159.9	1633.6	2793.5	2.930	5.966	265
270	55.030	0.001303	0.03562	1178.1	2593.7	1185.2	1604.5	2789.7	2.976	5.930	270
275	59.464	0.001317	0.03277	1203.0	2590.3	1210.9	1574.3	2785.2	3.022	5.894	275
280	64.166	0.001333	0.03015	1228.3	2586.4	1236.8	1543.0	2779.9	3.068	5.858	280
285	69.146	0.001349	0.02776	1253.9	2581.8	1263.2	1510.5	2773.7	3.114	5.821	285
290	74.418	0.001366	0.02555	1279.8	2576.5	1290.0	1476.7	2766.7	3.161	5.783	290
295	79.991	0.001384	0.02353	1306.1	2570.5	1317.2	1441.5	2758.7	3.208	5.745	295
300	85.879	0.001404	0.02166	1332.8	2563.6	1344.9	1404.7	2749.6	3.255	5.706	300
305	92.094	0.001425	0.01993	1360.1	2555.8	1373.2	1366.2	2739.4	3.302	5.666	305
310	98.650	0.001447	0.01833	1387.8	2547.1	1402.1	1325.8	2727.9	3.351	5.624	310
315	105.561	0.001472	0.01685	1416.2	2537.2	1431.7	1283.3	2715.0	3.399	5.582	315
320	112.843	0.001499	0.01547	1445.2	2526.0	1462.1	1238.5	2700.6	3.449	5.537	320
325	120.510	0.001528	0.01418	1475.0	2513.4	1493.4	1190.9	2684.3	3.500	5.491	325
330	128.581	0.001560	0.01298	1505.7	2499.2	1525.8	1140.2	2666.0	3.552	5.442	330
335	137.073	0.001597	0.01185	1537.5	2483.0	1559.4	1086.0	2645.4	3.605	5.391	335
340	146.007	0.001638	0.01078	1570.7	2464.5	1594.6	1027.4	2621.9	3.660	5.336	340
345	155.406	0.001685	0.00977	1605.4	2443.2	1631.6	963.4	2595.1	3.718	5.276	345
350	165.293	0.001741	0.00881	1642.3	2418.3	1671.1	892.7	2563.8	3.779	5.211	350
355	175.700	0.001808	0.00787	1682.1	2388.6	1713.9	812.9	2526.9	3.844	5.138	355
360	186.660	0.001895	0.00695	1726.2	2351.8	1761.5	720.0	2481.6	3.916	5.054	360
365	198.218	0.002015	0.00601	1777.2	2303.6	1817.2	605.5	2422.7	4.000	4.949	365
370	210.438	0.002217	0.00495	1844.5	2230.1	1891.2	443.1	2334.3	4.112	4.801	370
373.95	**220.640**	**0.00311**	**0.00311**	**2018.1**	**2018.1**	**2086.6**	**0.0**	**2086.6**	**4.410**	**4.410**	**373.95**

Specific enthalpy of water and steam

Pressure (bar)	0.1	0.5	1	5	10	20	40	60	80	100	150	200	220.64	250	300	400	500	1000
sat temp. (°C)	45.8	81.3	99.6	151.8	179.9	212.4	250.4	275.6	295.0	311.0	342.2	365.8	373.95	–	–	–	–	–
H (sat. liq.)	191.8	340.5	417.5	640.1	762.5	908.5	1087.5	1213.9	1317.2	1407.9	1610.3	1826.6	2086.6	–	–	–	–	–
H (sat. vap.)	2583.9	2645.2	2674.9	2748.1	2777.1	2798.3	2800.8	2784.6	2758.7	2725.5	2610.8	2412.1	2086.6	–	–	–	–	–
Temp. (°C)	**Specific enthalpy (kJ/kg)**																	
0.01	0	0.1	0.1	0.5	1	2	4.1	6.1	8.1	10.1	15.1	20.1	22.1	25	29.9	39.6	49.2	95.4
25	104.8	104.9	104.9	105.3	105.8	106.7	108.5	110.4	112.2	114.1	118.6	123.2	125.1	127.8	132.3	141.3	150.2	194.1
50	2592.0	209.4	209.4	209.8	210.2	211.1	212.8	214.5	216.2	217.9	222.2	226.5	228.3	230.8	235.1	243.6	252.0	293.9
75	2639.8	314.0	314.1	314.4	314.8	315.6	317.2	318.8	320.5	322.1	326.1	330.1	331.8	334.2	338.2	346.2	354.3	394.3
100	2687.5	2682.4	2675.8	419.5	419.8	420.6	422.1	423.6	425.1	426.6	430.4	434.2	435.7	438.0	441.7	449.3	456.9	495.1
125	2735.2	2731.5	2726.7	525.3	525.6	526.3	527.7	529.1	530.5	531.8	535.3	538.8	540.3	542.4	545.9	553.0	560.1	596.3
150	2783.0	2780.2	2776.6	632.2	632.5	633.1	634.4	635.6	636.9	638.1	641.3	644.4	645.8	647.7	650.9	657.4	664.0	697.9
175	2831.2	2828.9	2826.1	2801.4	741.1	741.6	742.7	743.7	744.8	745.9	748.6	751.4	752.6	754.2	757.1	763.0	768.9	800.2
200	2879.6	2877.8	2875.5	2855.8	2828.3	852.5	853.3	854.1	854.9	855.8	858.0	860.3	861.2	862.6	865.0	870.0	875.2	903.4
225	2928.4	2926.8	2924.9	2908.8	2887.0	2836.1	967.1	967.6	968.1	968.7	970.1	971.7	972.4	973.4	975.2	979.0	983.2	1007.6
250	2977.4	2976.1	2974.5	2961.0	2943.1	2903.2	1085.8	1085.7	1085.7	1085.8	1086.1	1086.7	1087.0	1087.4	1088.4	1090.7	1093.5	1113.1
275	3026.9	3025.8	3024.4	3012.9	2997.8	2965.2	2887.3	1210.9	1210.0	1209.3	1207.8	1206.7	1206.4	1206.0	1205.7	1205.8	1206.8	1220.2
300	3076.7	3075.8	3074.5	3064.6	3051.6	3024.2	2961.7	2885.5	2786.5	1343.3	1338.3	1334.4	1333.0	1331.3	1328.9	1325.6	1324.0	1329.1
325	3126.9	3126.1	3125.0	3116.3	3105.0	3081.5	3029.5	2969.5	2898.4	2810.3	1485.6	1475.2	1471.7	1467.3	1461.1	1452.2	1446.4	1440.3
350	3177.5	3176.8	3175.8	3168.1	3158.2	3137.7	3093.3	3043.9	2988.1	2924.0	2693.1	1646.0	1635.6	1623.9	1608.8	1588.8	1576.1	1554.0
375	3228.5	3227.9	3227.0	3220.1	3211.3	3193.2	3154.7	3112.8	3066.9	3016.3	2858.9	2602.6	2337.7	1849.4	1791.8	1742.6	1716.6	1670.8
400	3279.9	3279.3	3278.6	3272.3	3264.5	3248.3	3214.5	3178.2	3139.4	3097.4	2975.7	2816.9	2732.9	2578.6	2152.8	1931.4	1874.4	1791.1
425	3331.8	3331.2	3330.5	3324.9	3317.8	3303.3	3273.2	3241.4	3207.7	3172.0	3072.3	2953.0	2896.1	2805.0	2611.8	2199.0	2060.7	1915.7
450	3384.0	3383.5	3382.8	3377.7	3371.3	3358.2	3331.2	3302.9	3273.3	3242.3	3157.9	3061.7	3017.8	2950.6	2821.0	2511.8	2284.7	2044.7
475	3436.6	3436.2	3435.6	3430.9	3425.1	3413.2	3388.7	3363.4	3337.1	3309.7	3236.6	3155.8	3120.0	3066.2	2966.8	2740.2	2520.0	2178.5
500	3489.7	3489.3	3488.7	3484.5	3479.1	3468.2	3446.0	3423.1	3399.5	3375.1	3310.8	3241.2	3210.8	3165.9	3084.7	2906.5	2722.6	2316.2
550	3597.1	3596.8	3596.3	3592.7	3588.1	3579.0	3560.3	3541.3	3521.8	3502.0	3450.4	3396.1	3373.0	3339.2	3279.7	3154.4	3025.3	2595.9
600	3706.3	3706.0	3705.6	3702.5	3698.6	3690.7	3674.9	3658.7	3642.4	3625.8	3583.1	3539.0	3520.4	3493.5	3446.7	3350.4	3252.5	2865.1
650	3817.2	3816.9	3816.6	3813.9	3810.5	3803.8	3790.1	3776.2	3762.3	3748.1	3712.1	3675.3	3659.8	3637.7	3599.4	3521.6	3443.4	3110.5
700	3929.9	3929.7	3929.4	3927.0	3924.1	3918.2	3906.3	3894.3	3882.2	3870.0	3839.1	3807.8	3794.7	3776.0	3743.9	3679.1	3614.6	3330.7
750	4044.4	4044.2	4043.9	4041.8	4039.3	4034.1	4023.6	4013.2	4002.6	3992.0	3965.2	3938.1	3926.9	3910.9	3883.4	3828.4	3773.9	3530.5
800	4160.6	4160.4	4160.2	4158.4	4156.1	4151.5	4142.3	4133.1	4123.8	4114.5	4091.1	4067.5	4057.7	4043.8	4020.0	3972.6	3925.8	3715.3

Specific internal energy of water and steam

Pressure (bar)	**0.1**	**0.5**	**1**	**5**	**10**	**20**	**40**	**60**	**80**	**100**	**150**	**200**	**220.64**	**250**	**300**	**400**	**500**	**1000**
sat temp. (°C)	45.8	81.3	99.6	151.8	179.9	212.4	250.4	275.6	295.0	311.0	342.2	365.8	373.95	–	–	–	–	–
U (sat. liq.)	191.8	340.5	417.4	639.5	761.4	906.2	1082.5	1206.0	1306.1	1393.4	1585.4	1785.9	2018.1	–	–	–	–	–
U (sat. vap.)	2437.2	2483.2	2505.5	2560.7	2582.7	2599.1	2601.7	2589.9	2570.5	2545.2	2455.7	2294.8	2018.1	–	–	–	–	–
Temp. (°C)	**Specific internal energy (kJ/kg)**																	
0.01	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	–0.2
25	104.8	104.8	104.8	104.8	104.7	104.7	104.5	104.4	104.2	104.1	103.7	103.3	103.2	102.9	102.6	101.9	101.1	97.7
50	2443.3	209.3	209.3	209.3	209.2	209.0	208.7	208.4	208.2	207.9	207.1	206.4	206.2	205.8	205.1	203.7	202.5	196.6
75	2479.5	314.0	314.0	313.9	313.8	313.6	313.1	312.7	312.3	311.9	310.8	309.8	309.4	308.8	307.8	305.9	304.1	295.8
100	2515.5	2511.5	2506.2	418.9	418.8	418.5	417.9	417.4	416.8	416.2	414.8	413.5	412.9	412.2	410.9	408.4	405.9	395.1
125	2551.6	2548.7	2545.0	524.7	524.5	524.2	523.4	522.7	522.0	521.2	519.5	517.8	517.1	516.1	514.4	511.2	508.2	494.6
150	2587.9	2585.7	2582.9	631.6	631.4	630.9	630.0	629.1	628.2	627.3	625.1	622.9	622.0	620.8	618.7	614.8	611.0	594.3
175	2624.5	2622.8	2620.6	2601.6	740.0	739.4	738.2	737.0	735.9	734.8	732.0	729.3	728.2	726.7	724.1	719.3	714.6	694.4
200	2661.3	2660.0	2658.2	2643.3	2622.2	850.1	848.6	847.2	845.7	844.3	840.8	837.5	836.1	834.2	831.1	825.1	819.4	795.1
225	2698.5	2697.4	2695.9	2683.9	2667.3	2628.5	962.3	960.5	958.6	956.8	952.4	948.1	946.4	944.1	940.2	932.7	925.8	896.6
250	2736.1	2735.1	2733.9	2723.8	2710.4	2680.2	1080.8	1078.2	1075.8	1073.4	1067.6	1062.2	1060.0	1057.0	1052.0	1042.7	1034.2	999.1
275	2774.0	2773.2	2772.1	2763.6	2752.3	2727.8	2668.9	1203.0	1199.5	1196.2	1188.3	1181.0	1178.1	1174.2	1167.7	1155.9	1145.2	1102.7
300	2812.3	2811.6	2810.6	2803.2	2793.6	2773.2	2726.2	2668.4	2592.3	1329.4	1317.6	1307.1	1303.1	1297.6	1288.9	1273.3	1259.6	1207.6
325	2850.9	2850.3	2849.5	2843.0	2834.7	2817.2	2778.3	2733.1	2679.2	2611.6	1463.0	1445.8	1439.5	1431.2	1418.4	1396.7	1378.6	1314.3
350	2890.0	2889.4	2888.7	2883.0	2875.7	2860.5	2827.4	2790.4	2748.3	2699.6	2520.9	1612.7	1599.6	1583.9	1562.2	1529.3	1503.9	1422.8
375	2929.5	2929.0	2928.3	2923.2	2916.7	2903.3	2874.7	2843.4	2808.9	2770.7	2650.4	2449.1	2232.6	1799.9	1738.1	1677.0	1638.6	1533.6
400	2969.3	2968.9	2968.3	2963.7	2957.9	2945.9	2920.7	2893.7	2864.6	2833.1	2740.6	2617.9	2551.9	2428.5	2068.9	1854.9	1787.8	1646.8
425	3009.6	3009.2	3008.7	3004.5	2999.2	2988.5	2966.1	2942.4	2917.2	2890.4	2815.0	2723.5	2679.3	2607.8	2452.8	2097.5	1960.2	1762.9
450	3050.3	3049.9	3049.4	3045.6	3040.9	3031.1	3011.0	2989.9	2967.8	2944.5	2880.7	2807.2	2773.4	2721.2	2618.9	2364.2	2160.3	1881.9
475	3091.4	3091.0	3090.6	3087.1	3082.8	3073.9	3055.7	3036.7	3017.0	2996.5	2941.3	2879.7	2852.1	2810.5	2732.7	2549.7	2361.4	2003.6
500	3132.9	3132.6	3132.2	3129.0	3125.0	3116.9	3100.3	3083.1	3065.4	3047.0	2998.4	2945.3	2922.0	2887.3	2824.0	2681.6	2528.1	2126.9
550	3217.2	3217.0	3216.6	3213.9	3210.5	3203.6	3189.5	3175.2	3160.5	3145.4	3106.2	3064.7	3046.9	3020.8	2974.5	2875.0	2769.5	2371.0
600	3303.3	3303.1	3302.8	3300.4	3297.5	3291.5	3279.4	3267.2	3254.7	3242.0	3209.3	3175.3	3160.9	3140.0	3103.4	3026.8	2947.1	2597.9
650	3391.2	3391.0	3390.7	3388.6	3386.0	3380.8	3370.3	3359.6	3348.9	3337.9	3310.1	3281.4	3269.3	3251.9	3221.7	3159.5	3095.6	2798.9
700	3480.8	3480.6	3480.4	3478.5	3476.2	3471.6	3462.4	3453.0	3443.6	3434.0	3409.8	3385.1	3374.7	3359.9	3334.3	3282.0	3228.7	2976.1
750	3572.2	3572.0	3571.8	3570.2	3568.1	3564.0	3555.8	3547.5	3539.1	3530.7	3509.4	3487.7	3478.7	3465.8	3443.6	3398.6	3353.1	3135.2
800	3665.3	3665.2	3665.0	3663.6	3661.7	3658.0	3650.6	3643.2	3635.7	3628.2	3609.2	3590.1	3582.1	3570.7	3551.2	3511.8	3472.2	3281.7

Density of water and steam

Pressure (bar)	0.1	0.5	1	5	10	20	40	60	80	100	150	200	220.64	250	300	400	500	1000
sat temp. (°C)	45.8	81.3	99.6	151.8	179.9	212.4	250.4	275.6	295.0	311.0	342.2	365.8	373.95	–	–	–	–	–
ρ (sat. liq.)	990.1	970.9	958.8	914.9	887.3	849.6	798.7	758.2	722.5	688.7	603.5	490.7	322	–	–	–	–	–
ρ (sat. vap.)	0.0681	0.309	0.592	2.668	5.144	10.04	20.09	30.82	42.5	55.5	96.7	170.6	322	–	–	–	–	–
Temp. (°C)	**Density (kg/m^3)**																	
0.01	999.8	999.8	999.8	1000.0	1000.3	1000.8	1001.8	1002.8	1003.8	1004.8	1007.3	1009.7	1010.7	1012.2	1014.5	1019.2	1023.8	1045.3
25	997.0	997.0	997.0	997.2	997.5	997.9	998.8	999.7	1000.6	1001.5	1003.7	1005.8	1006.7	1008.0	1010.1	1014.3	1018.4	1037.9
50	0.0673	988.0	988.0	988.2	988.4	988.9	989.7	990.6	991.5	992.3	994.4	996.5	997.4	998.6	1000.7	1004.7	1008.7	1027.4
75	0.0624	974.8	974.8	975.0	975.2	975.7	976.6	977.5	978.3	979.2	981.3	983.5	984.4	985.6	987.7	991.8	995.8	1014.6
100	0.0582	0.293	0.590	958.5	958.8	959.2	960.2	961.1	962.0	962.9	965.2	967.4	968.4	969.6	971.8	976.1	980.3	999.8
125	0.0545	0.274	0.550	939.2	939.4	939.9	940.9	941.9	942.9	943.9	946.4	948.8	949.8	951.2	953.5	958.1	962.5	983.1
150	0.0512	0.257	0.516	917.0	917.3	917.9	919.0	920.1	921.2	922.3	925.0	927.7	928.8	930.3	932.9	937.9	942.7	964.8
175	0.0484	0.243	0.487	2.503	892.4	893.0	894.3	895.6	896.8	898.1	901.1	904.1	905.4	907.1	909.9	915.5	920.9	945.0
200	0.0458	0.230	0.460	2.353	4.850	865.0	866.5	868.0	869.5	870.9	874.5	878.0	879.4	881.3	884.6	890.9	897.0	923.7
225	0.0435	0.218	0.437	2.223	4.550	9.63	835.1	836.9	838.7	840.4	844.7	848.8	850.5	852.8	856.6	864.0	870.9	901.0
250	0.0414	0.207	0.416	2.108	4.300	8.97	798.9	801.2	803.5	805.7	811.0	816.1	818.1	820.9	825.6	834.3	842.4	876.7
275	0.0395	0.198	0.396	2.006	4.070	8.43	18.31	759.1	762.2	765.1	772.2	778.7	781.3	784.8	790.6	801.4	811.1	850.8
300	0.0378	0.189	0.379	1.913	3.880	7.97	16.99	27.63	41.2	715.3	725.6	734.7	738.2	743.0	750.7	764.4	776.5	823.2
325	0.0362	0.181	0.363	1.830	3.700	7.57	15.93	25.39	36.5	50.3	664.9	679.8	685.3	692.4	703.4	722.0	737.6	793.7
350	0.0348	0.174	0.348	1.754	3.540	7.21	15.04	23.67	33.4	44.6	87.1	600.6	612.0	625.5	643.9	671.9	693.2	762.3
375	0.0334	0.167	0.335	1.684	3.390	6.90	14.28	22.27	31.0	40.7	71.9	130.3	210.0	505.5	558.2	609.3	641.2	728.8
400	0.0322	0.161	0.322	1.620	3.260	6.61	13.62	21.09	29.1	37.8	63.8	100.5	121.9	166.5	357.4	523.3	577.8	692.9
425	0.0310	0.155	0.311	1.561	3.140	6.35	13.03	20.07	27.5	35.5	58.3	87.1	101.8	126.8	188.7	394.1	497.7	654.7
450	0.0300	0.150	0.300	1.506	3.030	6.11	12.49	19.17	26.2	33.6	54.1	78.6	90.3	109.0	148.4	270.9	402.0	614.2
475	0.0290	0.145	0.290	1.454	2.920	5.90	12.01	18.37	25.0	31.9	50.8	72.4	82.4	97.8	128.1	210.0	315.2	571.7
500	0.0280	0.140	0.280	1.407	2.820	5.69	11.57	17.65	23.9	30.5	48.0	67.6	76.4	89.7	115.1	177.8	257.1	528.3
550	0.0263	0.132	0.263	1.320	2.650	5.33	10.79	16.39	22.1	28.0	43.6	60.3	67.7	78.5	98.3	143.2	195.4	444.6
600	0.0248	0.124	0.248	1.244	2.490	5.01	10.12	15.32	20.6	26.1	40.1	55.0	61.4	70.7	87.4	123.6	163.7	374.2
650	0.0235	0.117	0.235	1.176	2.360	4.73	9.53	14.40	19.4	24.4	37.3	50.8	56.5	64.8	79.4	110.5	143.7	321.0
700	0.0223	0.111	0.223	1.115	2.230	4.48	9.01	13.60	18.2	22.9	34.9	47.3	52.5	60.1	73.2	100.7	129.6	282.0
750	0.0212	0.106	0.212	1.060	2.120	4.25	8.55	12.88	17.3	21.7	32.9	44.4	49.2	56.2	68.2	93.1	118.8	253.0
800	0.0202	0.101	0.202	1.010	2.020	4.05	8.14	12.25	16.4	20.6	31.1	41.9	46.4	52.8	64.0	86.8	110.2	230.6

AMMONIA PROPERTIES

Saturation Values							Superheat (T–T^{sat})			
							50°C		100°C	
Temp. (°C)	P^{sat} (bar)	V_{vap} (m^3/kg)	H_{liq} (kJ/kg)	H_{vap} (kJ/kg)	S_{liq} (kJ/kg.K)	S_{vap} (kJ/kg.K)	H_{vap} (kJ/kg)	S_{vap} (kJ/kg.K)	H_{vap} (kJ/kg)	S_{vap} (kJ/kg.K)
–50	0.4089	2.625	–44.4	1373.3	–0.194	6.159	1479.8	6.592	1585.9	6.948
–45	0.5454	2.005	–22.3	1381.6	–0.096	6.057	1489.3	6.486	1596.1	6.839
–40†	0.7177	1.552	0†	1390.0	0†	5.962	1498.6	6.387	1606.3	6.736
–35	0.9322	1.216	22.3	1397.9	0.095	5.872	1507.9	6.293	1616.3	6.639
–30	1.196	0.9633	44.7	1405.6	0.188	5.785	1517.0	6.203	1626.3	6.547
–28	1.317	0.8809	53.6	1408.5	0.224	5.751	1520.7	6.169	1630.3	6.512
–26	1.447	0.8058	62.6	1411.4	0.261	5.718	1524.3	6.135	1634.2	6.477
–24	1.588	0.7389	71.7	1414.3	0.297	5.686	1527.9	6.103	1638.2	6.444
–22	1.740	0.6783	80.8	1417.3	0.333	5.655	1531.4	6.071	1642.2	6.411
–20	1.902	0.6237	89.8	1420.0	0.368	5.623	1534.8	6.039	1646.0	6.379
–18	2.077	0.5743	98.8	1422.7	0.404	5.593	1538.2	6.008	1650.0	6.347
–16	2.265	0.5296	107.9	1425.3	0.440	5.563	1541.7	5.978	1653.8	6.316
–14	2.465	0.4890	117.0	1427.9	0.475	5.533	1545.1	5.948	1657.7	6.286
–12	2.680	0.4521	126.2	1430.5	0.510	5.504	1548.5	5.919	1661.5	6.256
–10	2.908	0.4185	135.4	1433.0	0.544	5.475	1551.7	5.891	1665.3	6.227
–8	3.153	0.3879	144.5	1435.3	0.579	5.447	1554.9	5.863	1669.0	6.199
–6	3.413	0.3599	153.6	1437.6	0.613	5.419	1558.2	5.836	1672.8	6.171
–4	3.691	0.3344	162.8	1439.9	0.647	5.392	1561.4	5.808	1676.4	6.143
–2	3.983	0.3110	172.0	1442.2	0.681	5.365	1564.6	5.782	1680.1	6.116
0	4.295	0.2895	181.2	1444.4	0.715	5.340	1567.8	5.756	1683.9	6.090
2	4.625	0.2699	190.4	1446.5	0.749	5.314	1570.9	5.731	1687.5	6.065
4	4.975	0.2517	199.7	1448.5	0.782	5.288	1574.0	5.706	1691.2	6.040
6	5.346	0.2351	209.1	1450.6	0.816	5.263	1577.0	5.682	1694.9	6.015
8	5.736	0.2198	218.5	1452.5	0.849	5.238	1580.1	5.658	1698.4	5.991
10	6.149	0.2056	227.8	1454.3	0.881	5.213	1583.1	5.634	1702.2	5.967
12	6.585	0.1926	237.2	1456.1	0.914	5.189	1586.0	5.611	1705.7	5.943
14	7.045	0.1805	246.6	1457.8	0.947	5.165	1588.9	5.588	1709.1	5.920
16	7.529	0.1693	256.0	1459.5	0.979	5.141	1591.7	5.565	1712.5	5.898
18	8.035	0.1590	265.5	1461.1	1.012	5.118	1594.4	5.543	1715.9	5.876
20	8.570	0.1494	275.1	1462.6	1.044	5.095	1597.2	5.521	1719.3	5.854
22	9.134	0.1405	284.6	1463.9	1.076	5.072	1600.0	5.499	1722.8	5.832
24	9.722	0.1322	294.1	1465.2	1.108	5.049	1602.7	5.478	1726.3	5.811
26	10.34	0.1245	303.7	1466.5	1.140	5.027	1605.3	5.458	1729.6	5.790
28	10.99	0.1173	313.4	1467.8	1.172	5.005	1608.0	5.437	1732.7	5.770
30	11.67	0.1106	323.1	1468.9	1.204	4.984	1610.5	5.417	1735.9	5.750
32	12.37	0.1044	332.8	1469.9	1.235	4.962	1613.0	5.397	1739.3	5.731
34	13.11	0.0986	342.5	1470.8	1.267	4.940	1615.4	5.378	1742.6	5.711
36	13.89	0.0931	352.3	1471.8	1.298	4.919	1617.8	5.358	1745.7	5.692
38	14.70	0.0880	362.1	1472.6	1.329	4.898	1620.1	5.340	1748.7	5.674
40	15.54	0.0833	371.9	1473.3	1.360	4.877	1622.4	5.321	1751.9	5.655
42	16.42	0.0788	381.8	1473.8	1.391	4.856	1624.6	5.302	1755.0	5.637
44	17.34	0.0746	391.8	1474.2	1.422	4.835	1626.8	5.284	1758.0	5.619
46	18.30	0.0706	401.8	1474.5	1.453	4.814	1629.0	5.266	1761.0	5.602
48	19.29	0.0670	411.9	1474.7	1.484	4.793	1631.1	5.248	1764.0	5.584
50	20.33	0.0635	421.9	1474.7	1.515	4.773	1633.1	5.230	1766.8	5.567

† The arbitrary datum chosen for this table is that saturated liquid ammonia at −40 °C has enthalpy $H = 0$ and entropy $S = 0$

BESSEL FUNCTIONS

x	J_0	Y_0	I_0	K_0	J_1	Y_1	I_1	K_1
0.0	1.000	$-\infty$	1.000	$+\infty$	0.000	$-\infty$	0.000	$+\infty$
0.2	0.990	–1.081	1.010	1.753	0.100	–3.324	0.101	4.776
0.4	0.960	–0.606	1.040	1.115	0.196	–1.781	0.204	2.184
0.6	0.912	–0.309	1.092	0.778	0.287	–1.260	0.314	1.303
0.8	0.846	–0.087	1.167	0.565	0.369	–0.978	0.433	0.862
1.0	0.765	0.088	1.266	0.421	0.440	–0.781	0.565	0.602
1.2	0.671	0.228	1.394	0.319	0.498	–0.621	0.715	0.435
1.4	0.567	0.338	1.553	0.244	0.542	–0.479	0.886	0.321
1.6	0.455	0.420	1.750	0.188	0.570	–0.348	1.085	0.241
1.8	0.340	0.477	1.990	0.146	0.582	–0.224	1.317	0.183
2.0	0.224	0.510	2.280	0.114	0.577	–0.107	1.591	0.140
2.2	0.110	0.521	2.629	0.089	0.556	0.001	1.914	0.108
2.4	0.003	0.510	3.049	0.070	0.520	0.100	2.298	0.084
2.6	–0.097	0.481	3.553	0.055	0.471	0.188	2.755	0.065
2.8	–0.185	0.436	4.157	0.044	0.410	0.264	3.301	0.051
3.0	–0.260	0.377	4.881	0.035	0.339	0.325	3.953	0.040
3.2	–0.320	0.307	5.747	0.028	0.261	0.371	4.734	0.032
3.4	–0.364	0.230	6.785	0.022	0.179	0.401	5.670	0.025
3.6	–0.392	0.148	8.028	0.017	0.095	0.415	6.793	0.020
3.8	–0.403	0.065	9.517	0.014	0.013	0.414	8.140	0.016
4.0	–0.397	–0.017	11.302	0.011	–0.066	0.398	9.759	0.012

GAUSSIAN ERROR FUNCTION

$$\mathbf{erf}(x) = 1 - \mathbf{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \qquad \frac{d}{dy}[\mathbf{erf}(x)] = \frac{2}{\sqrt{\pi}} e^{-x^2} \frac{dx}{dy}$$

x	0.0000	0.1000	0.2000	0.3000	0.4000	0.5000
erf x	0.0000	0.1125	0.2227	0.3286	0.4284	0.5205
x	0.6000	0.7000	0.8000	0.9000	1.0000	1.1000
erf x	0.6039	0.6778	0.7421	0.7969	0.8427	0.8802
x	1.2000	1.3000	1.4000	1.5000	1.6000	1.7000
erf x	0.9103	0.9340	0.9523	0.9661	0.9763	0.9838
x	1.8000	1.9000	2.0000	∞		
erf x	0.9891	0.9928	0.9953	1.0		